Higher-Order Threshold Implementations

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- A countermeasure against Higher-Order Differential Power Analysis (HO-DPA)

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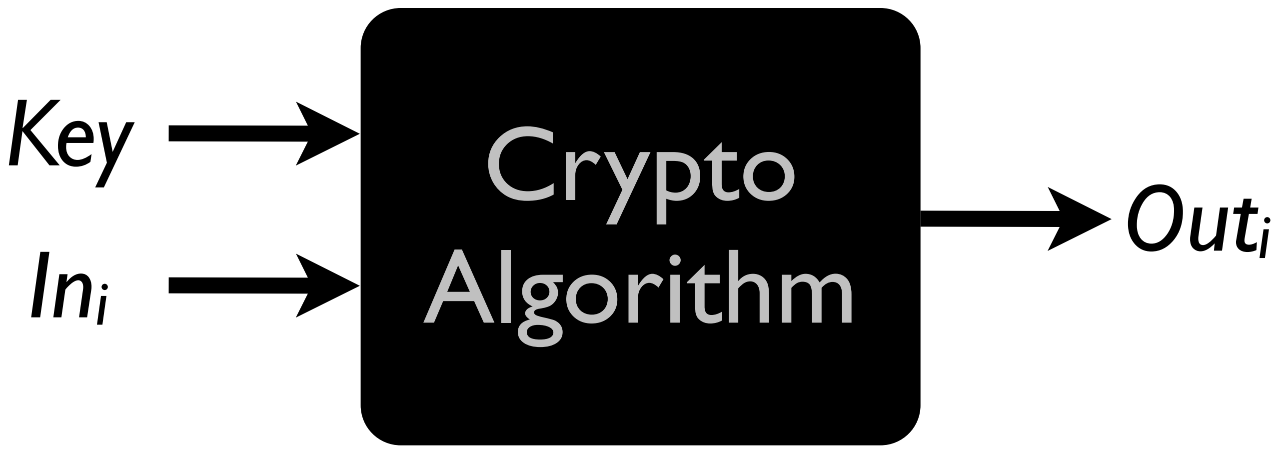
- A countermeasure against Higher-Order Differential Power Analysis (HO-DPA)
- Ideas from secret sharing and multi-party computation
- Can be applied to any algorithm
- Independent of technology, library, etc.
- Efficient
- Application to KATAN-32

Background

Differential Power Analysis
&
Its Countermeasures

Background - DPA



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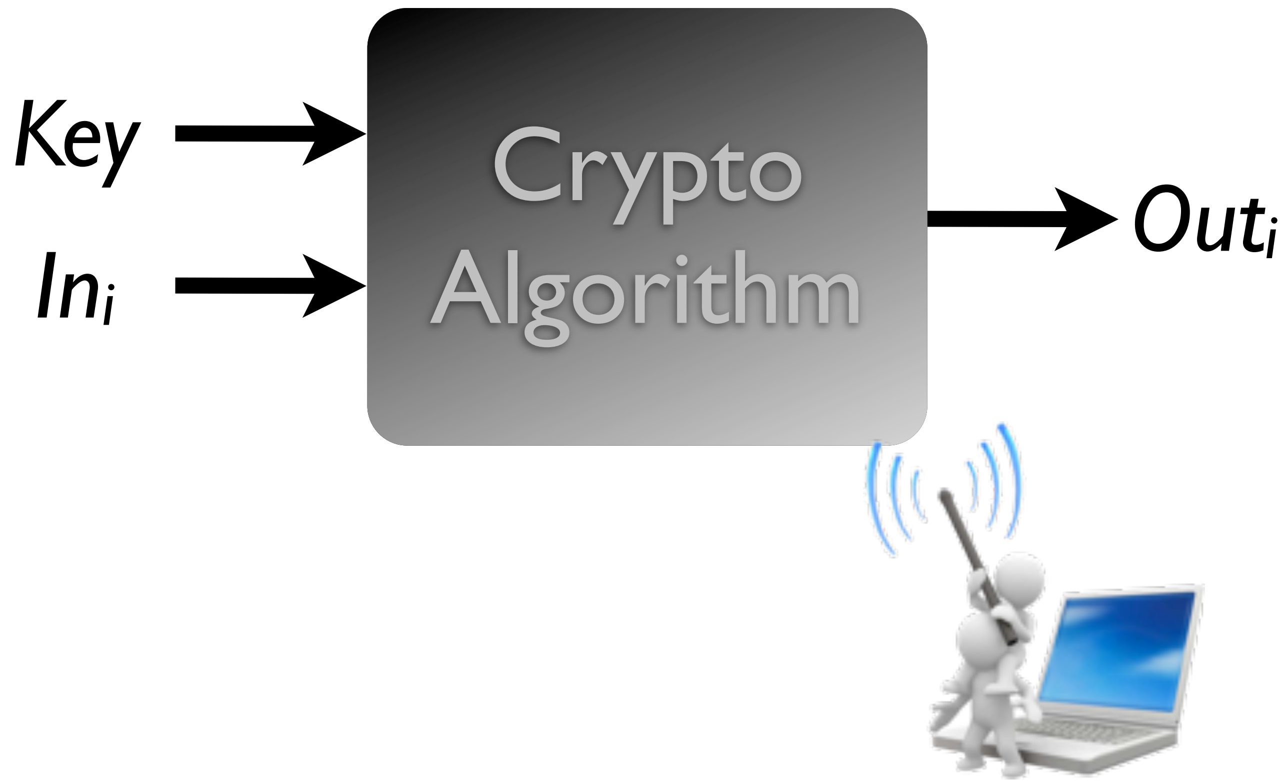

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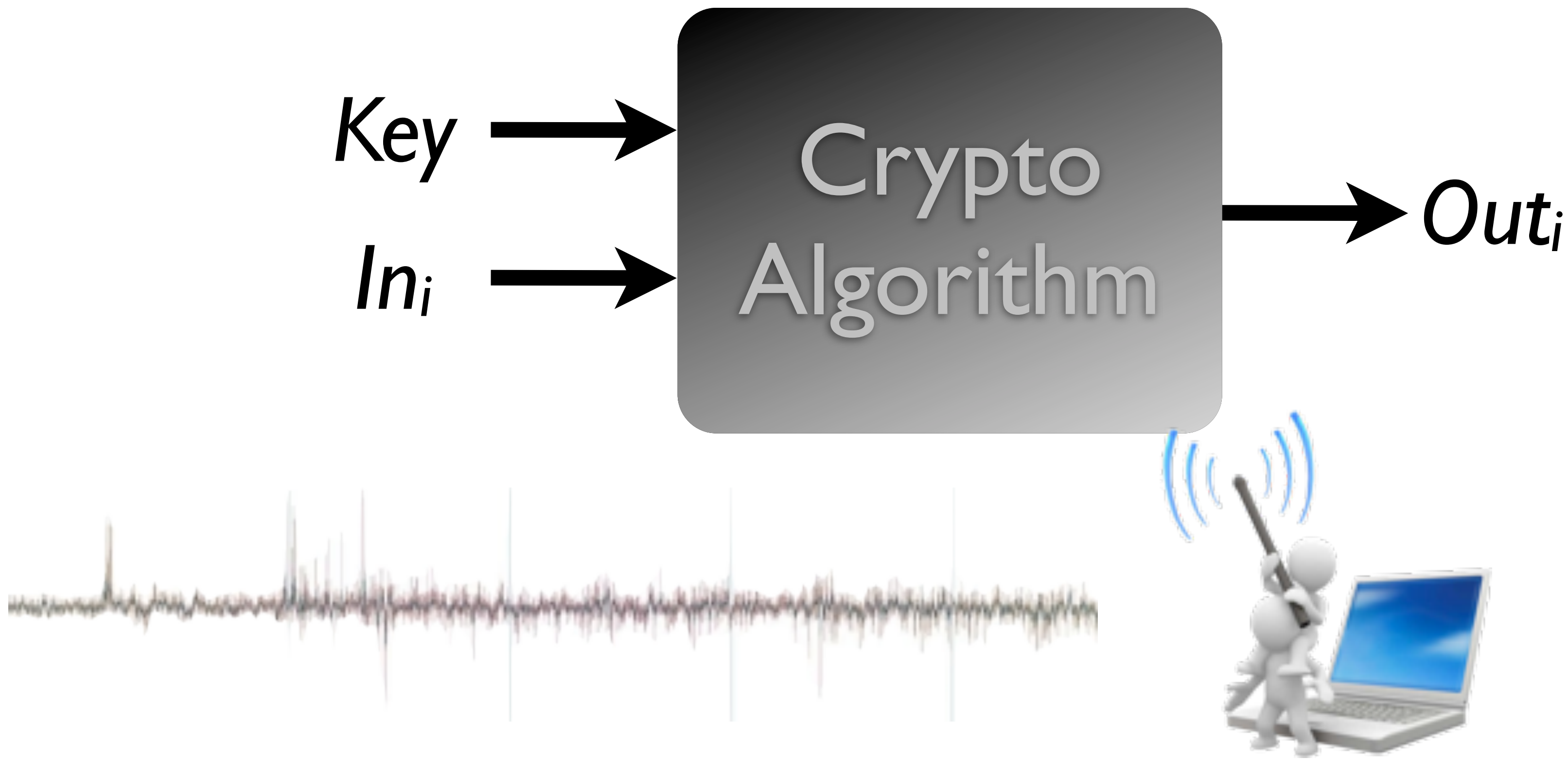

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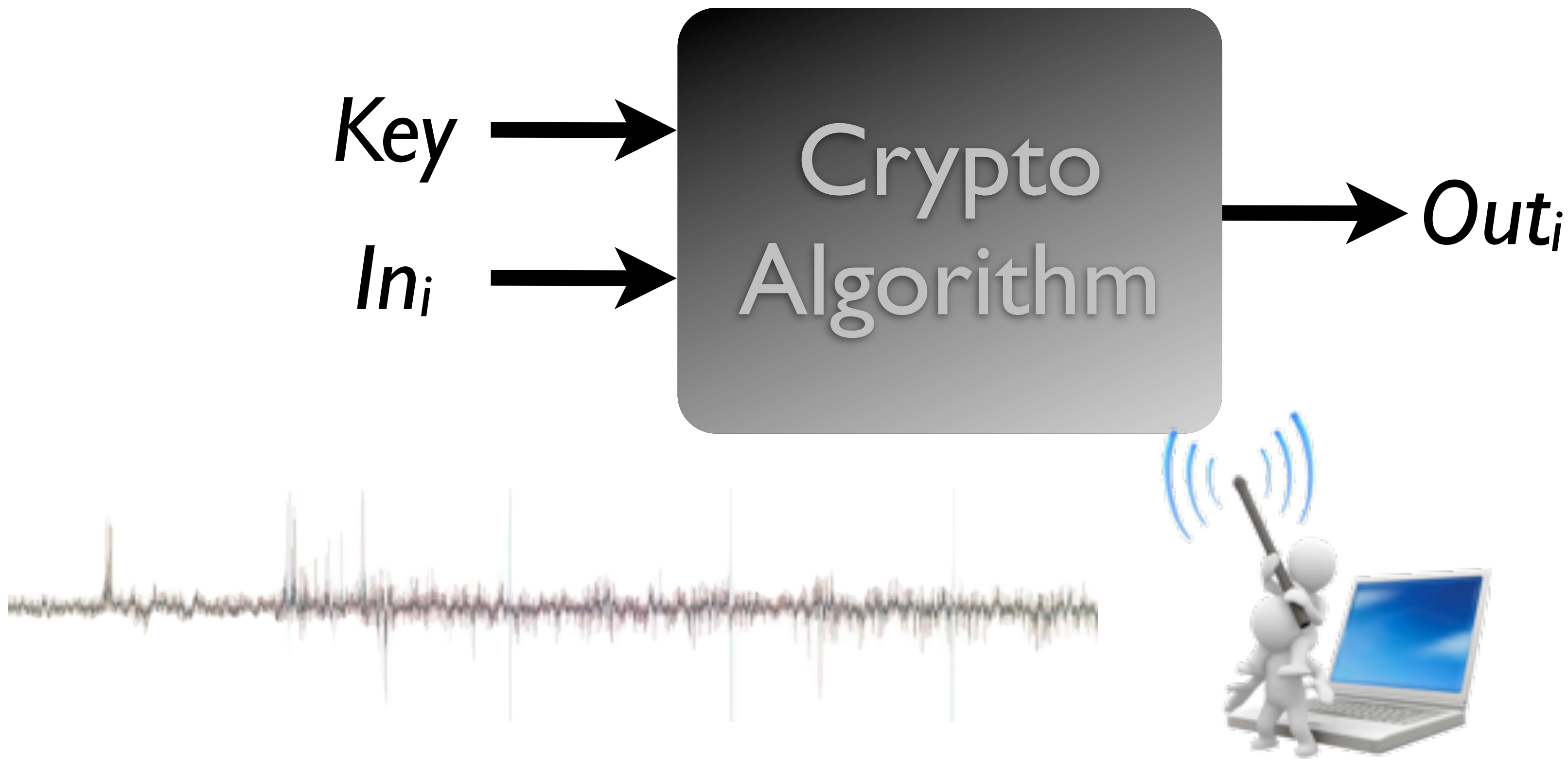

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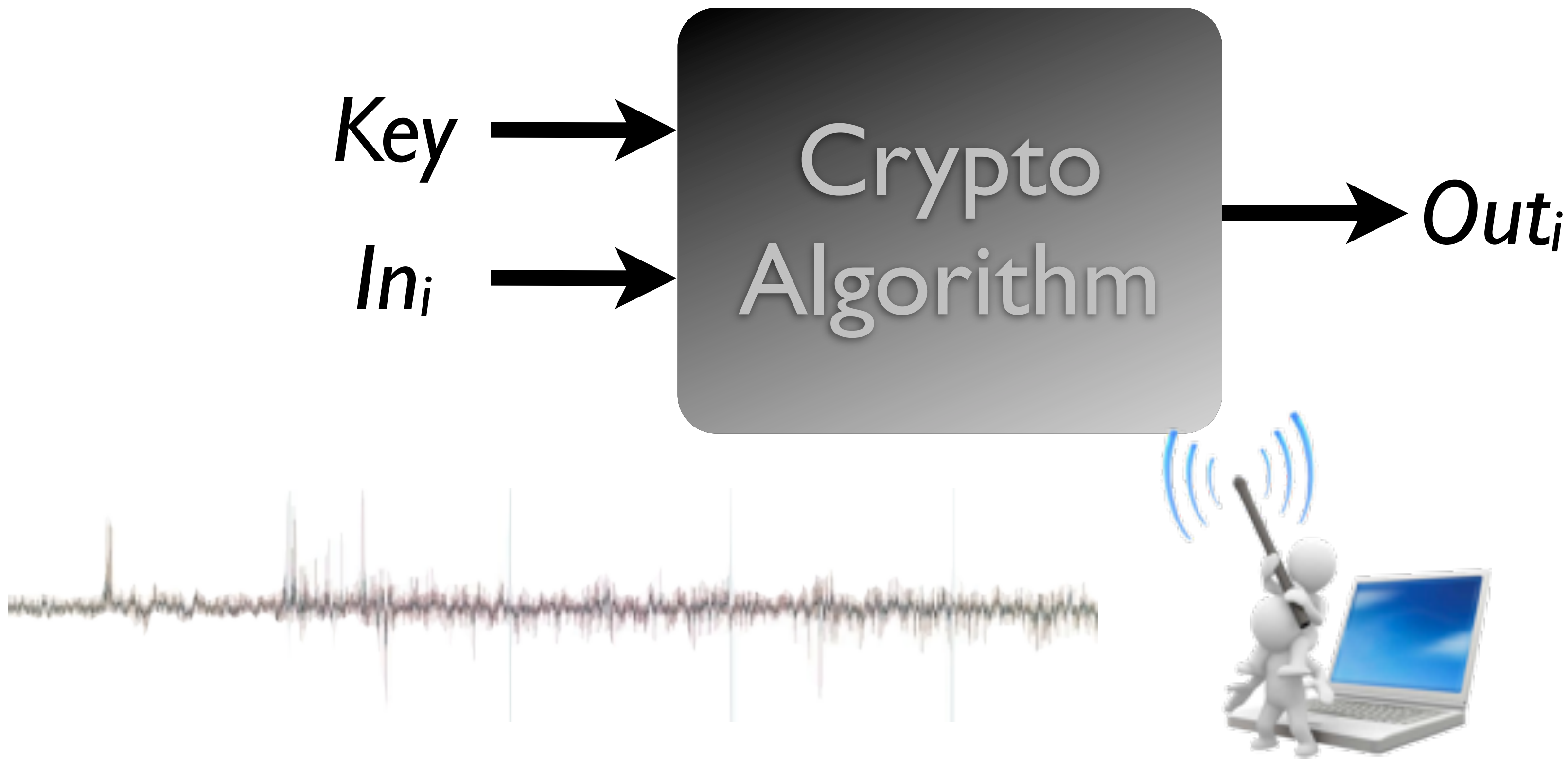

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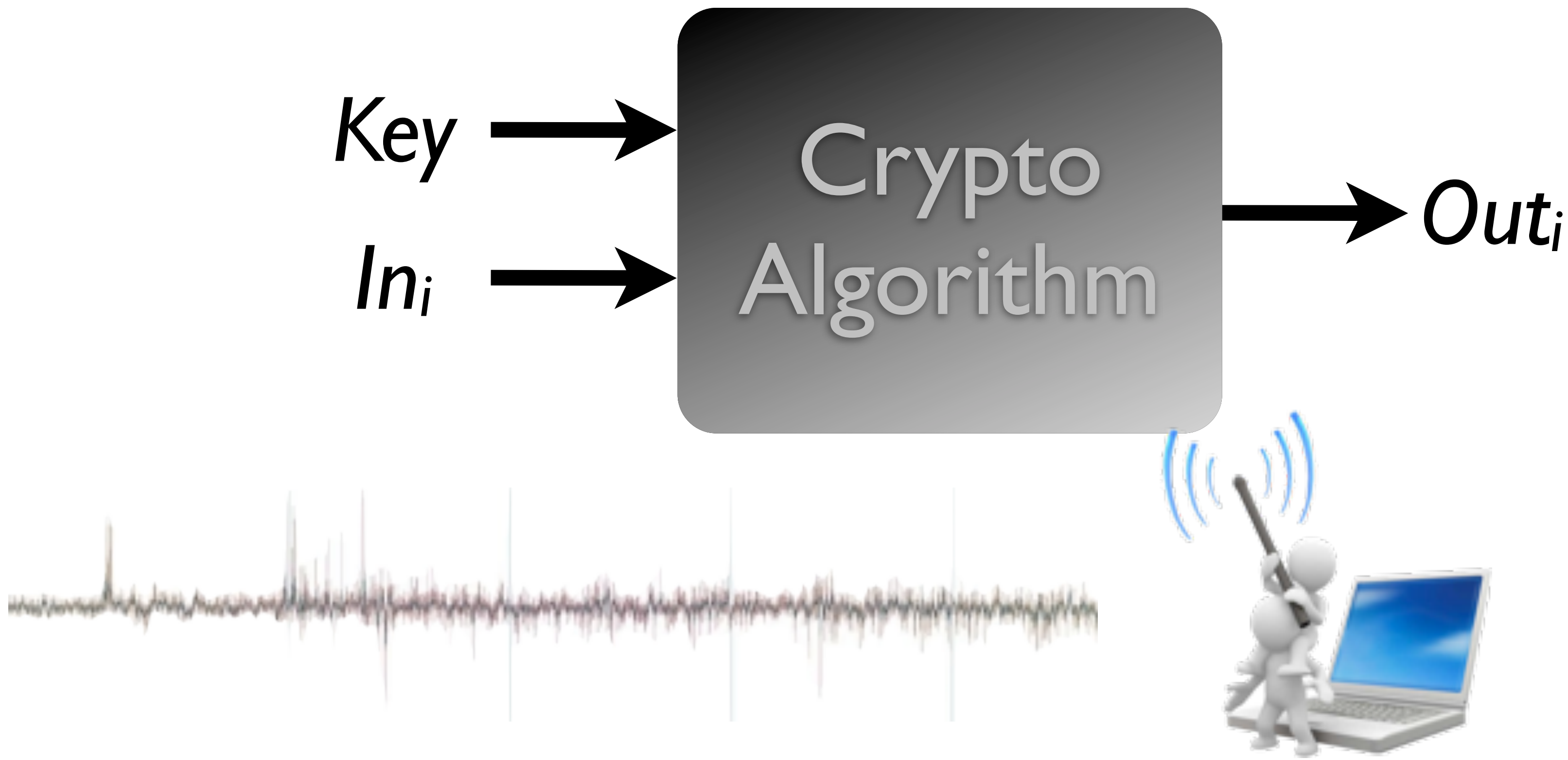

DPA focuses on finding the secret using the relation between

- Instantaneous power consumption
- And intermediate results of the algorithm

Background - DPA Countermeasures

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- Limiting the usage of key

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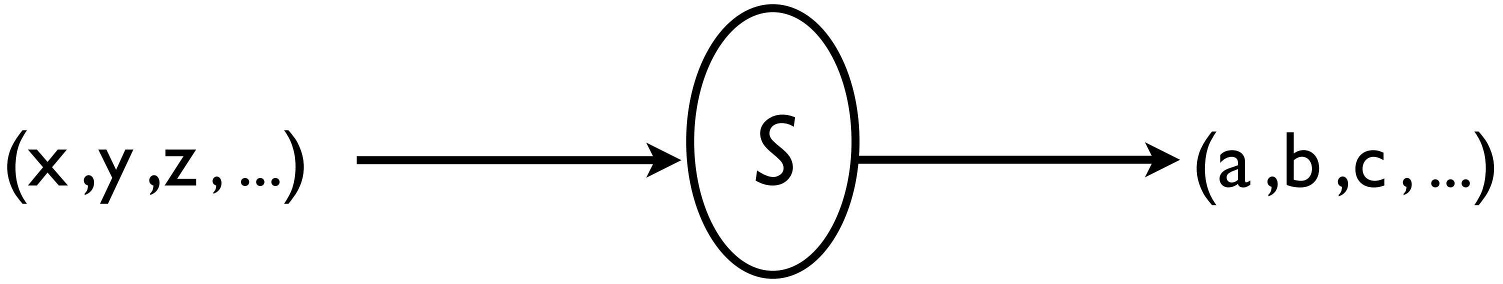
  e.g. Masking

Background - Boolean Masking



Background - Boolean Masking

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$(x_1, y_1, z_1, \ldots)$

Background - Boolean Masking

$$(x_1, y_1, z_1, \ldots)$$

$$\oplus$$

$$(x_2, y_2, z_2, \ldots)$$

$$=$$

$$(x, y, z, \ldots)$$

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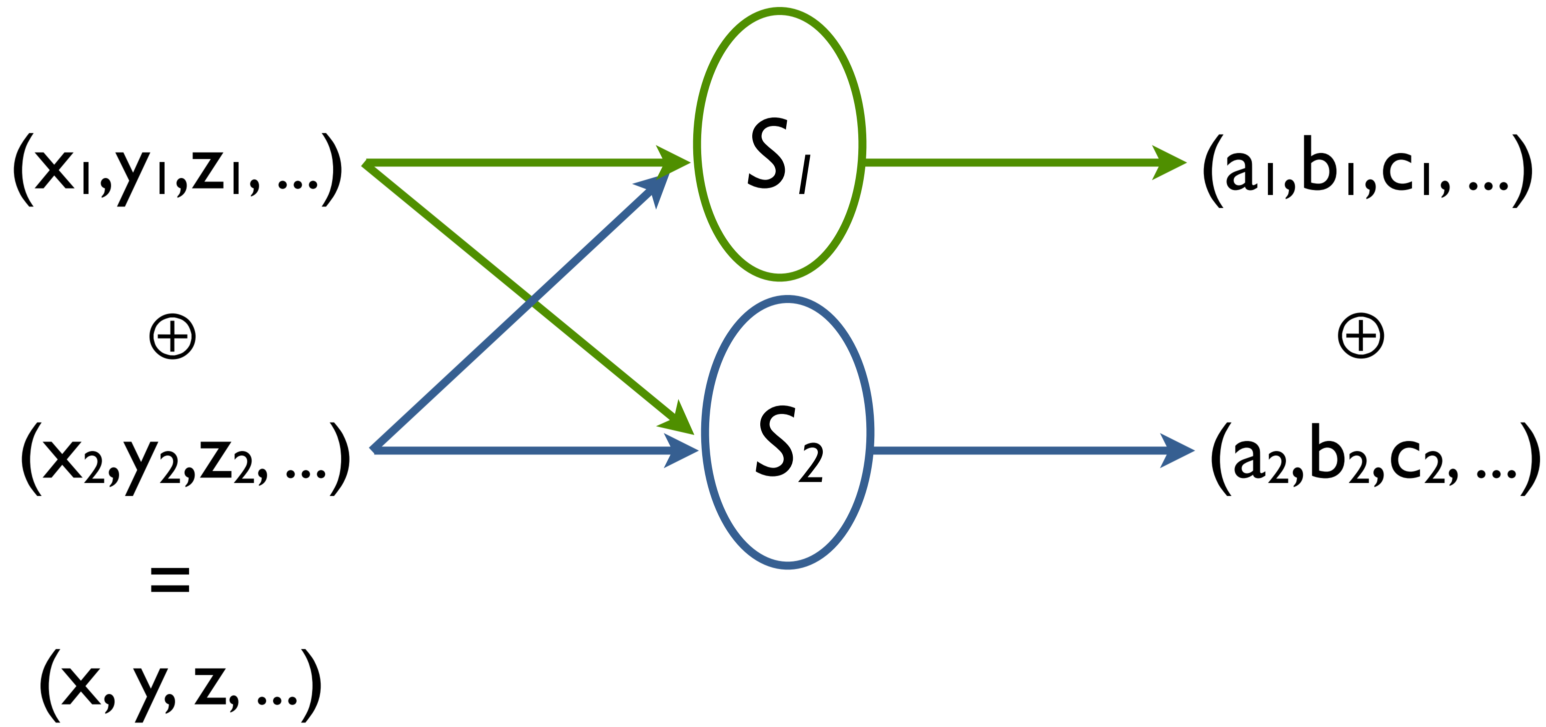

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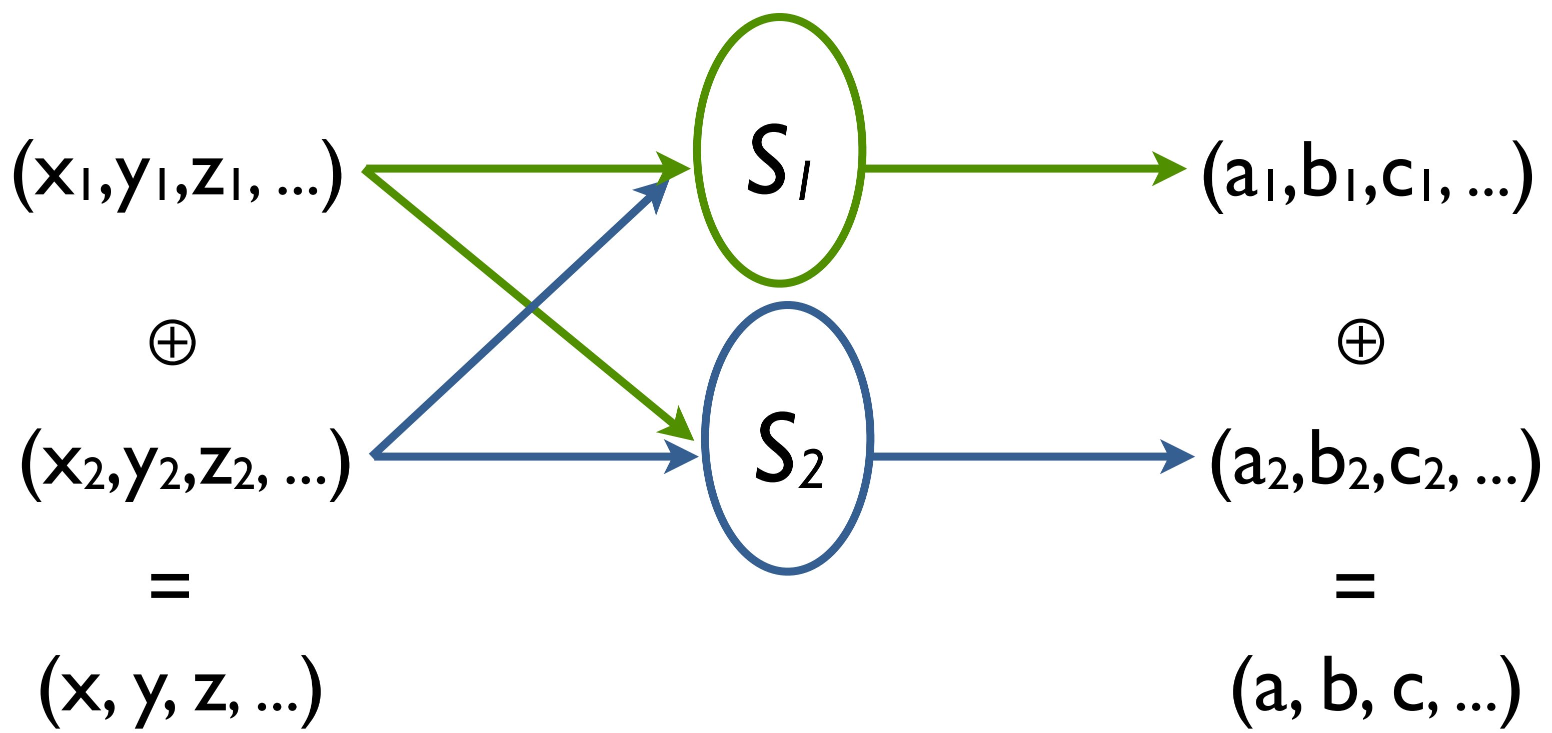

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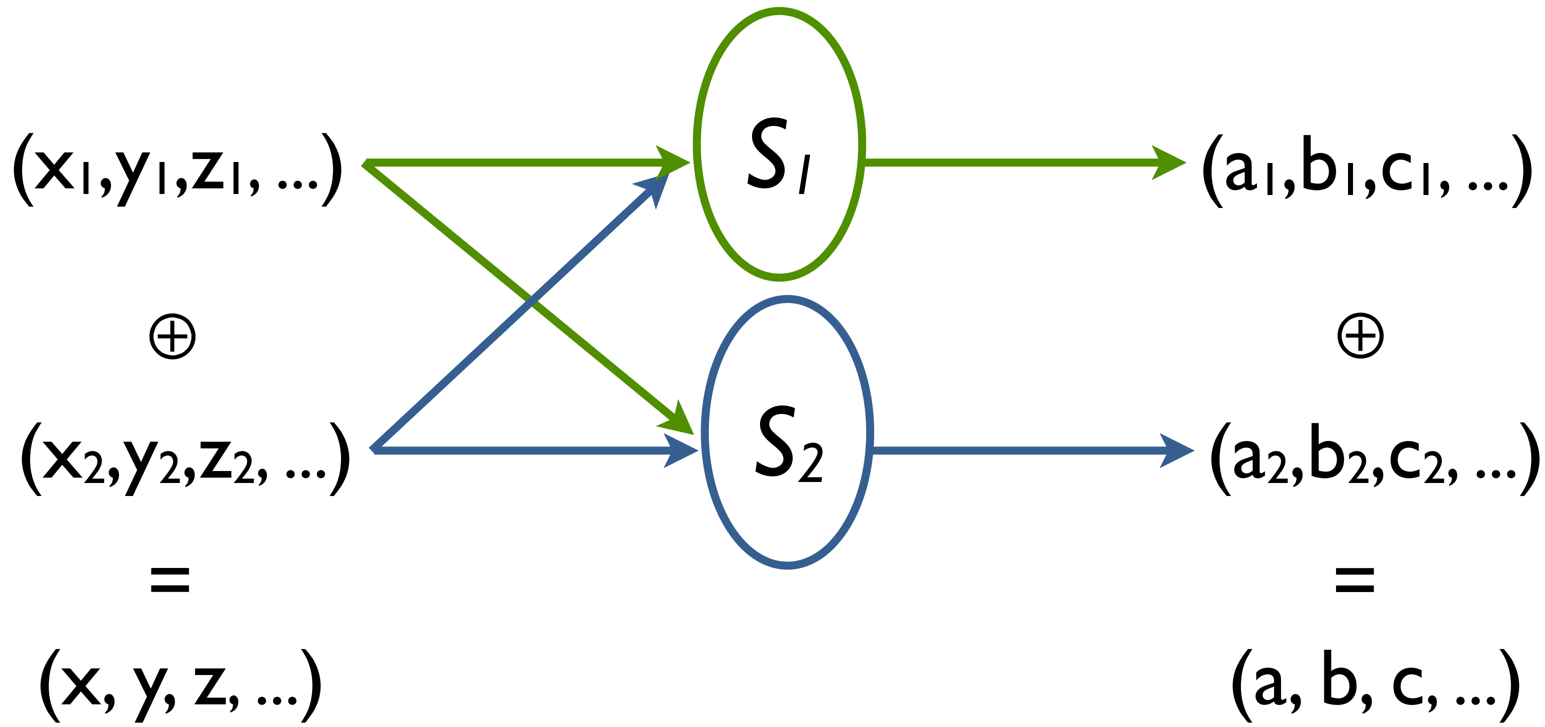

Random input/output shares ➡ Random intermediate values

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d^{th}-order DPA $\rightleftarrows$ d probing model

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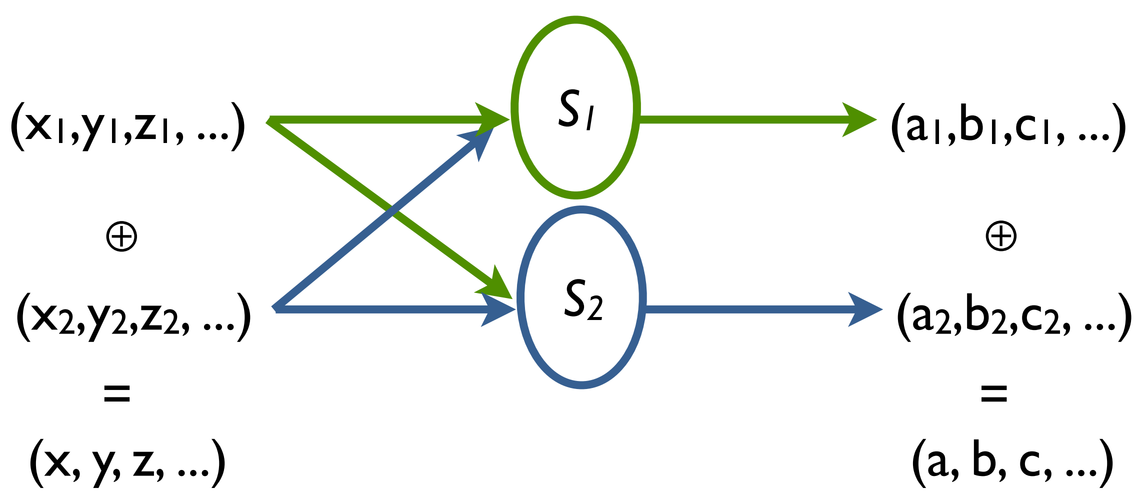

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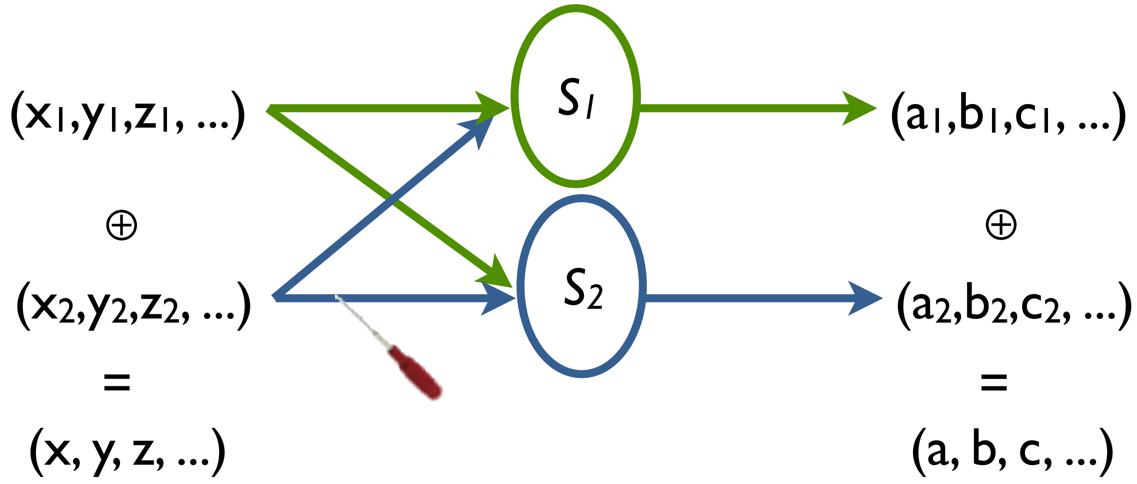

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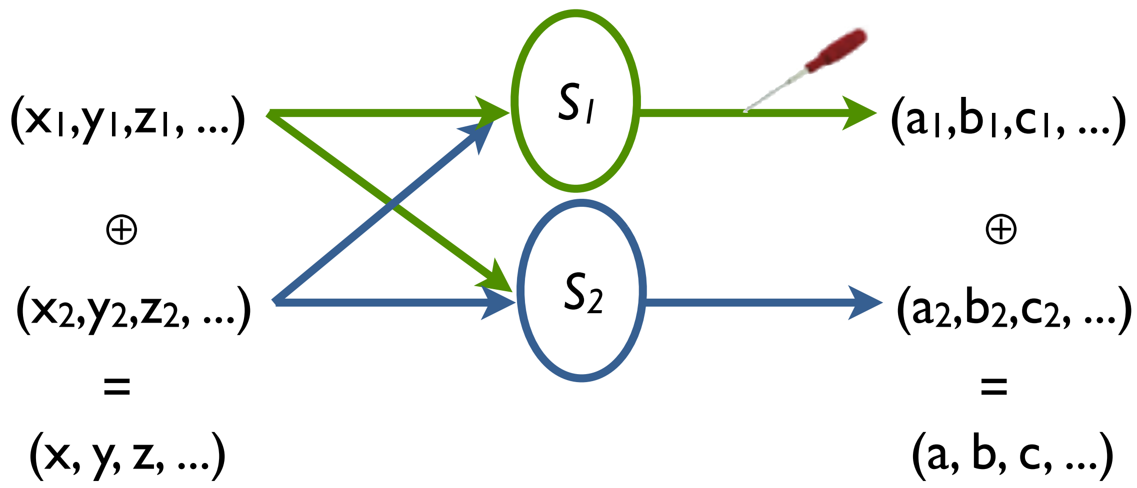

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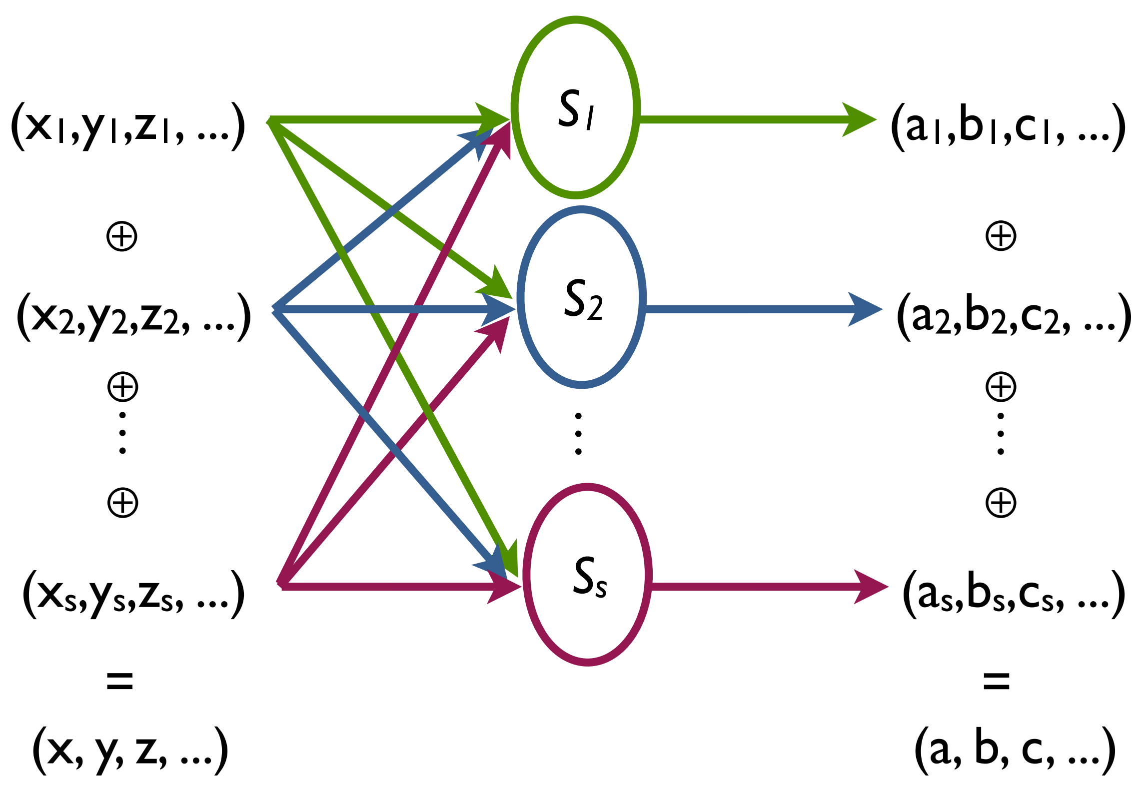

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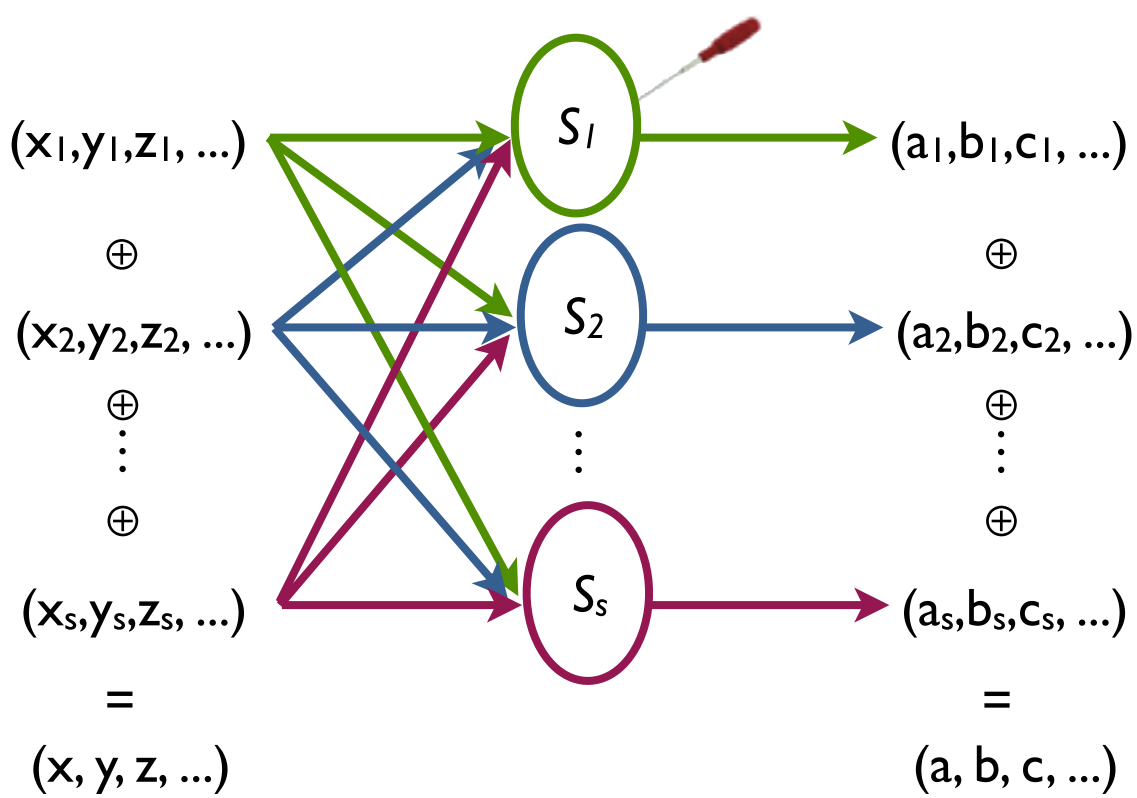

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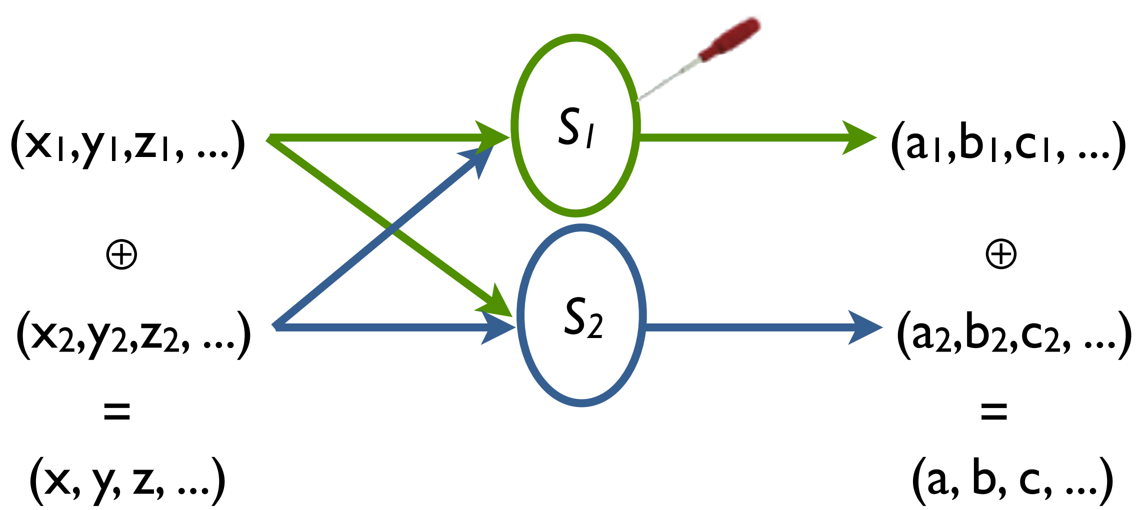

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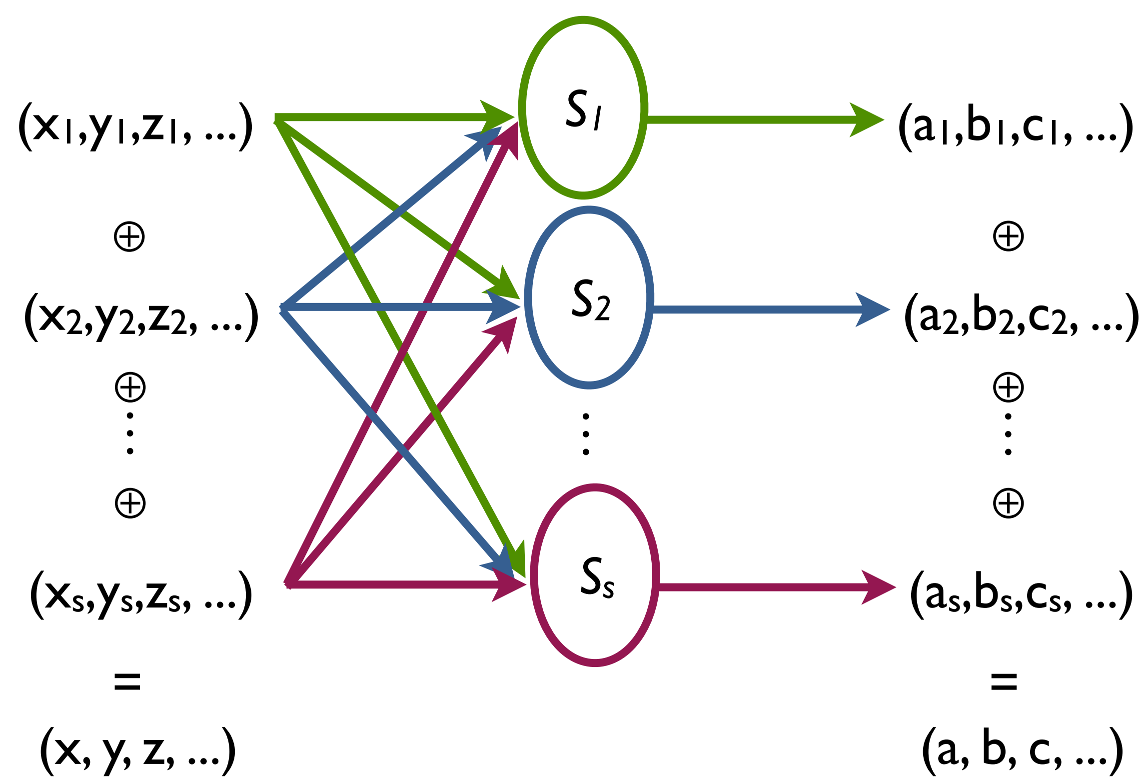

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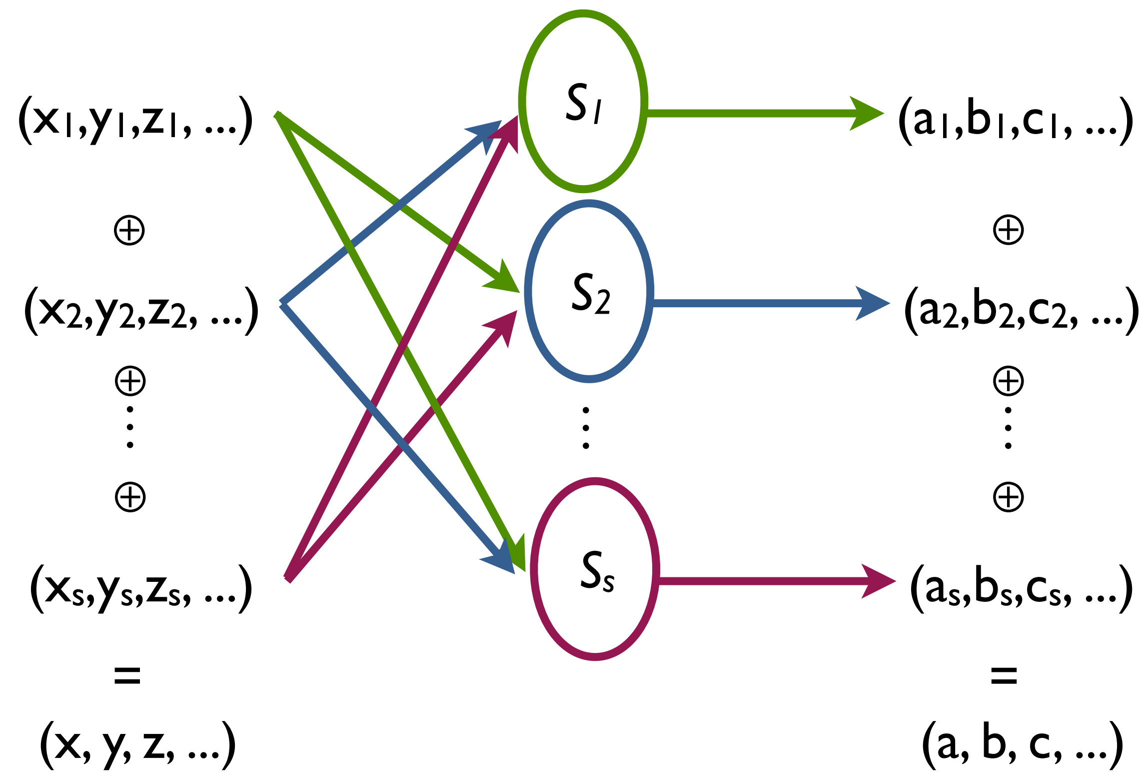

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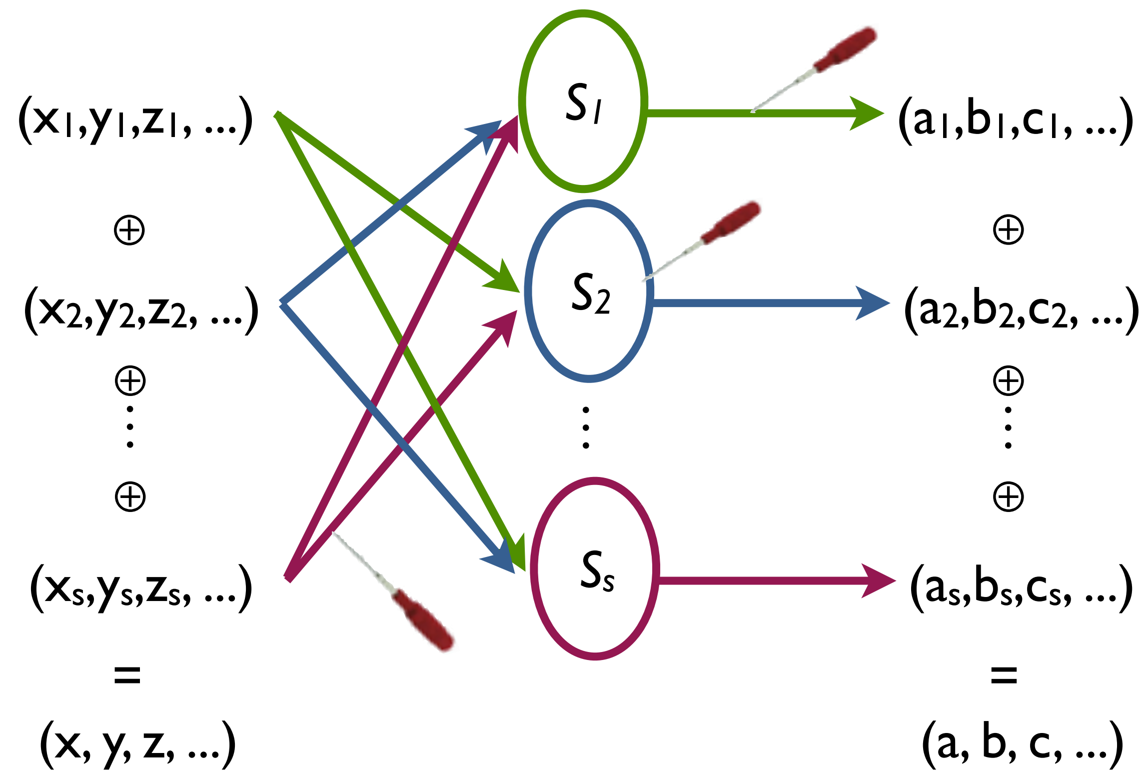

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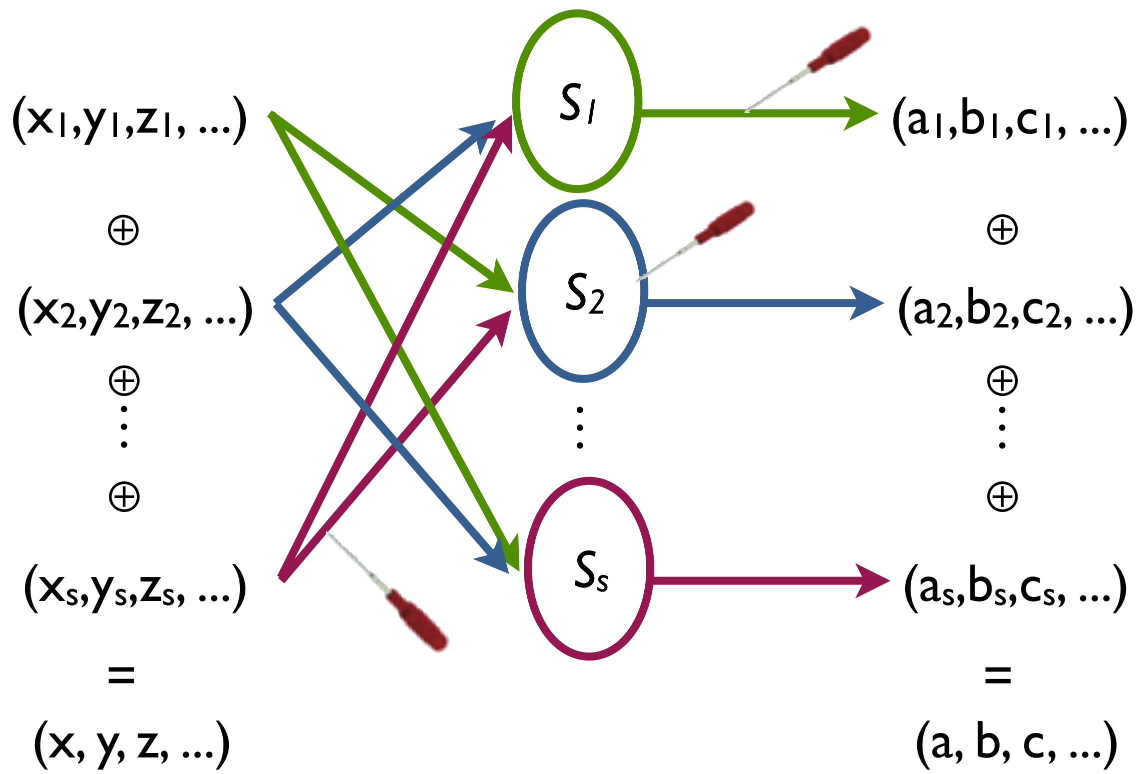

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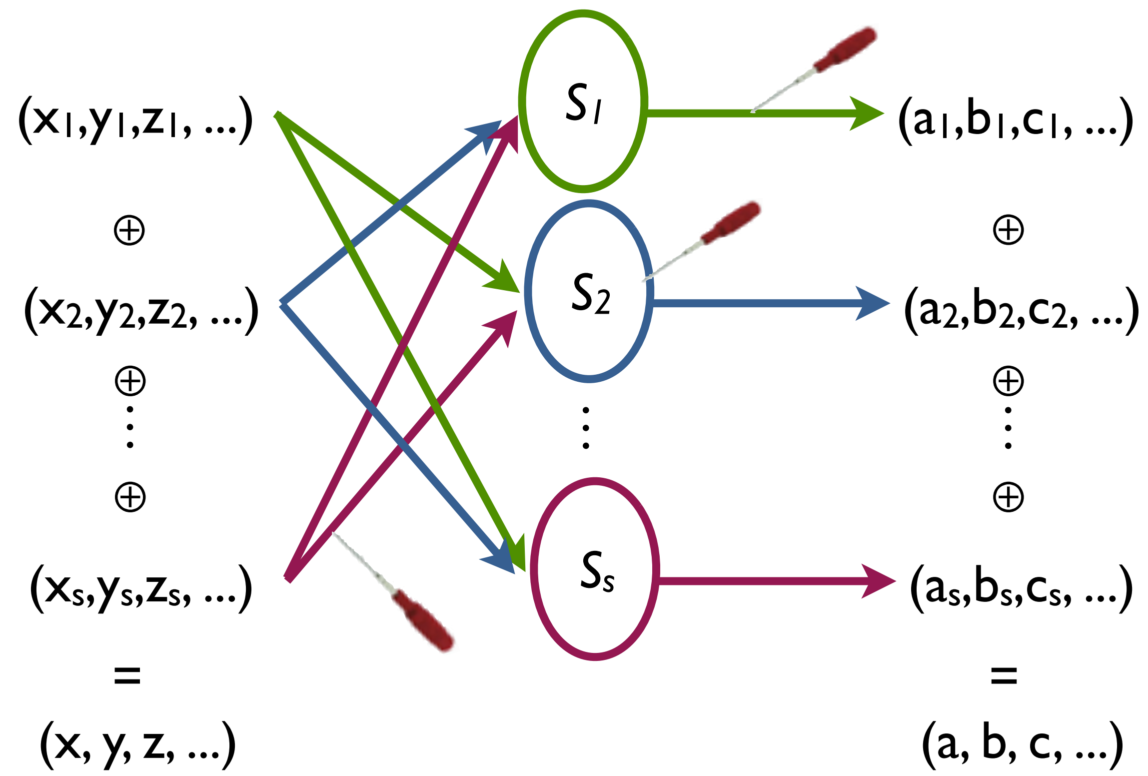

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- (*d=1*) Threshold Implementations (ICICS'06)

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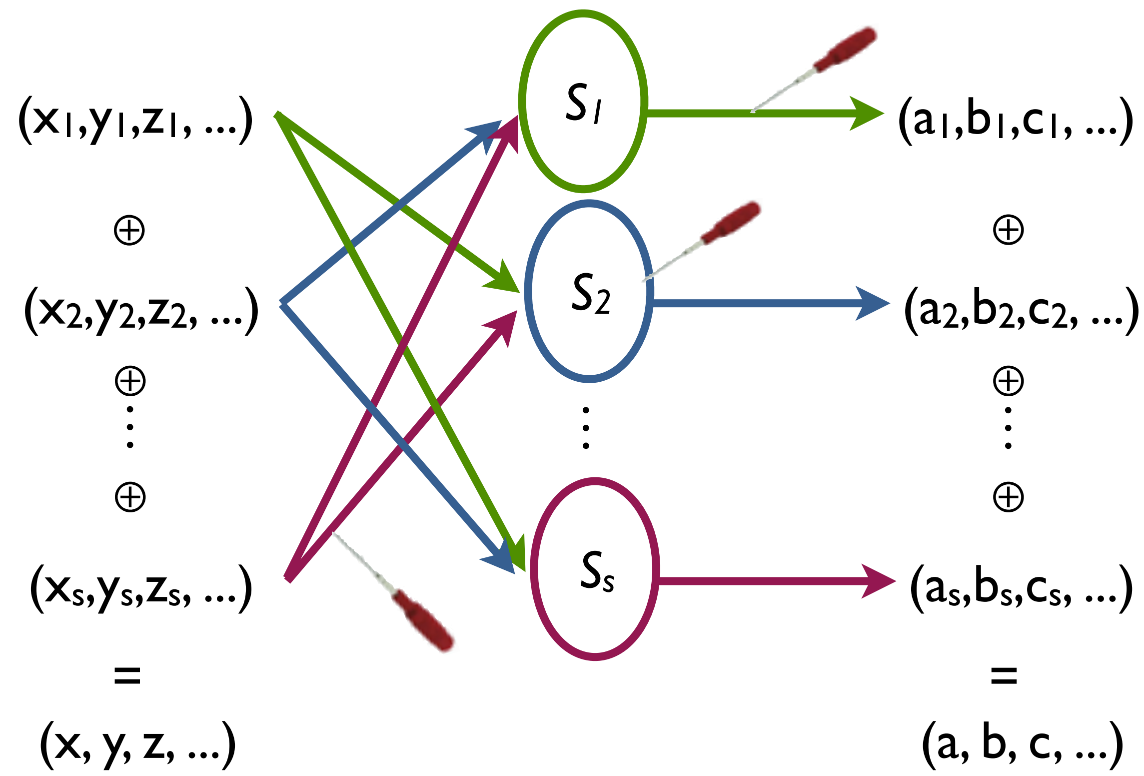

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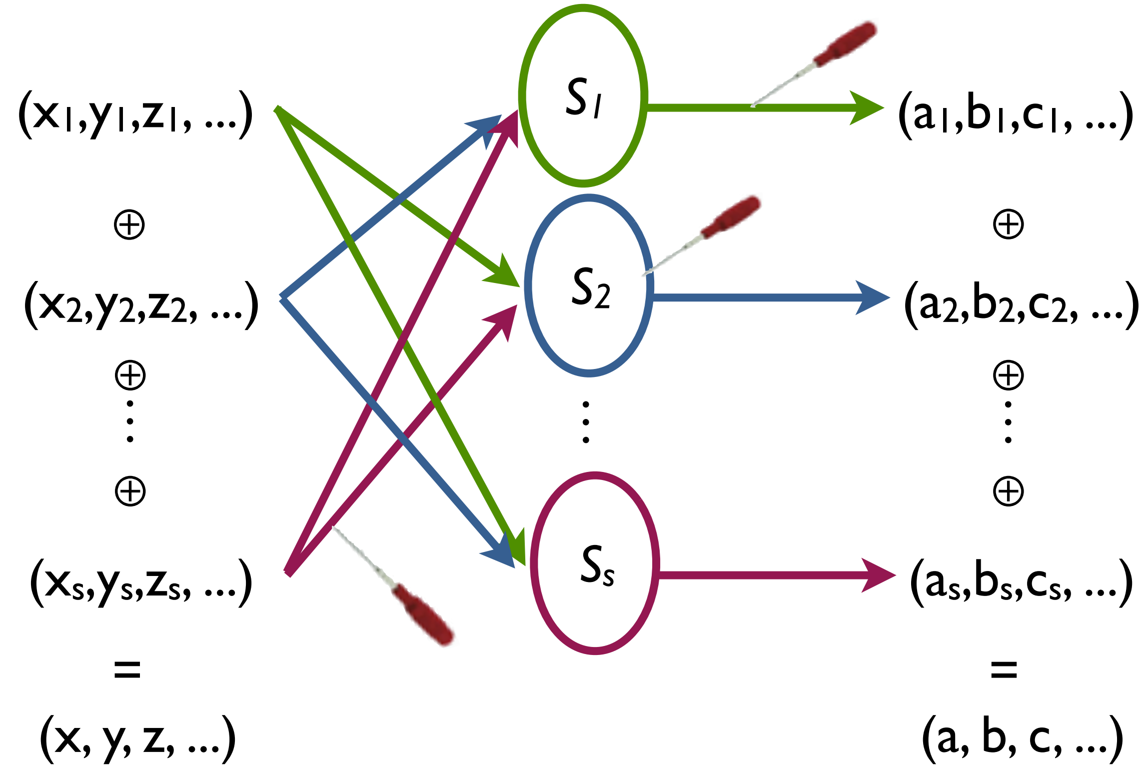

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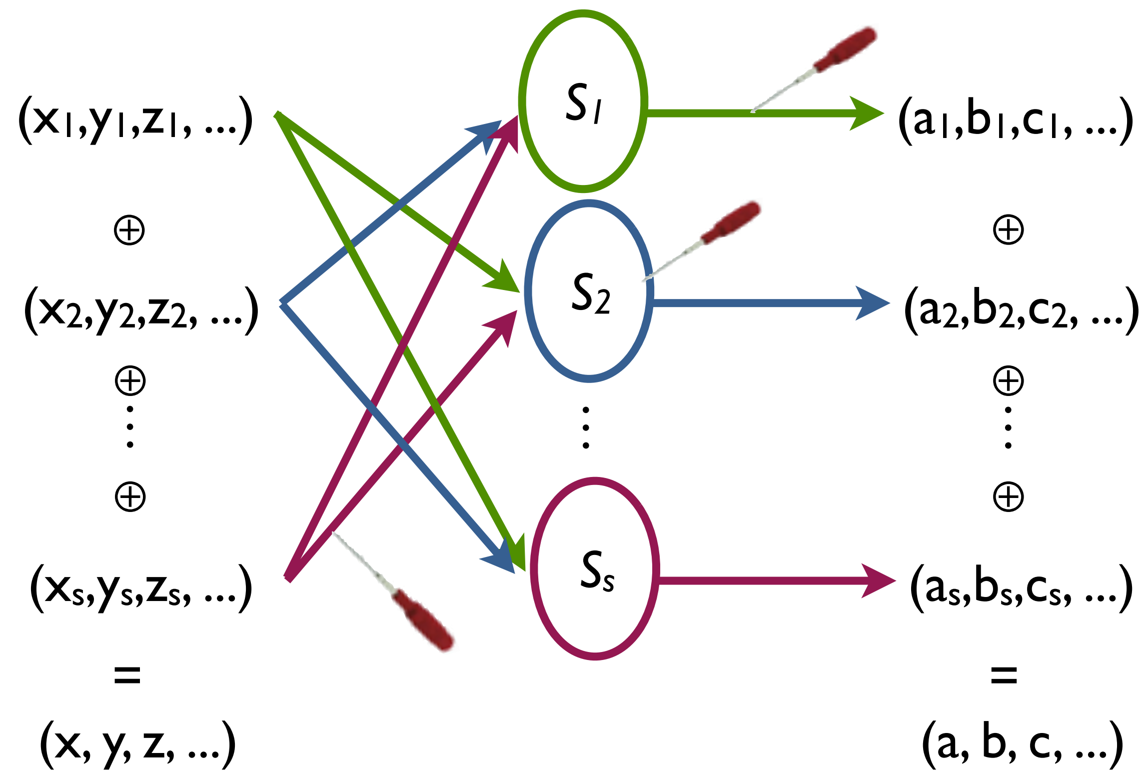

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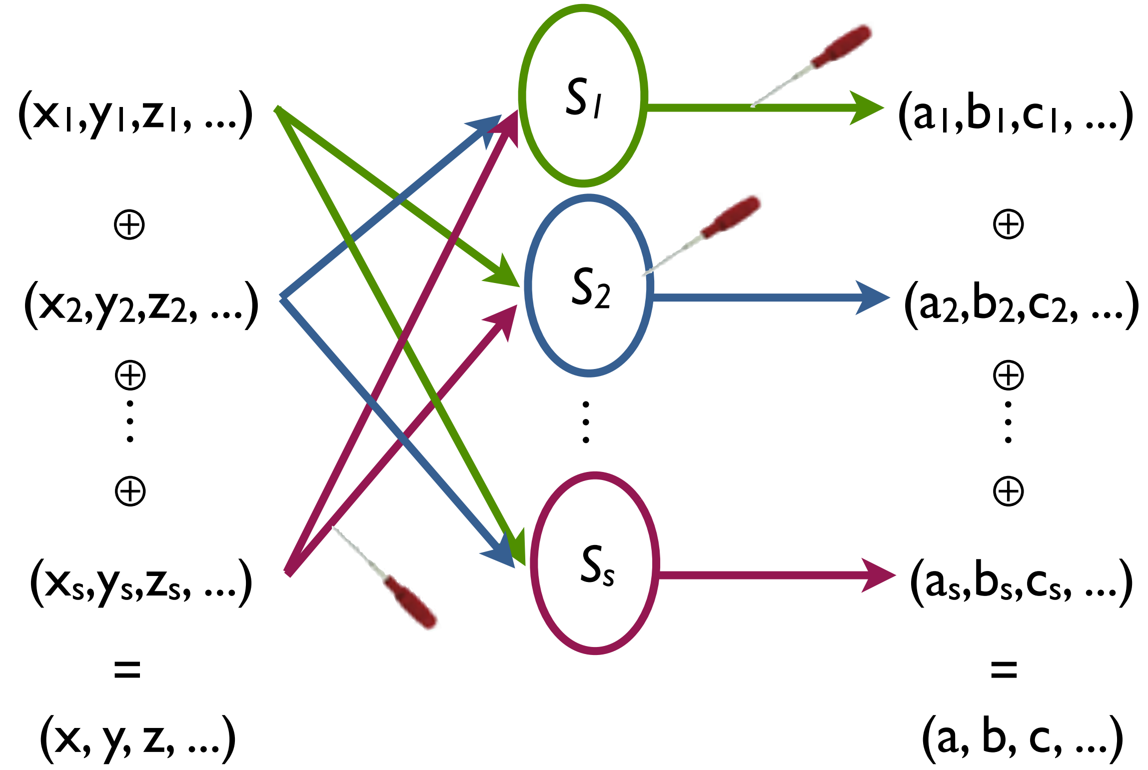
  #shares > 2d

Higher-Order Threshold Implementations

Properties
&
Requirements

Threshold Implementations (*d*=*1*)



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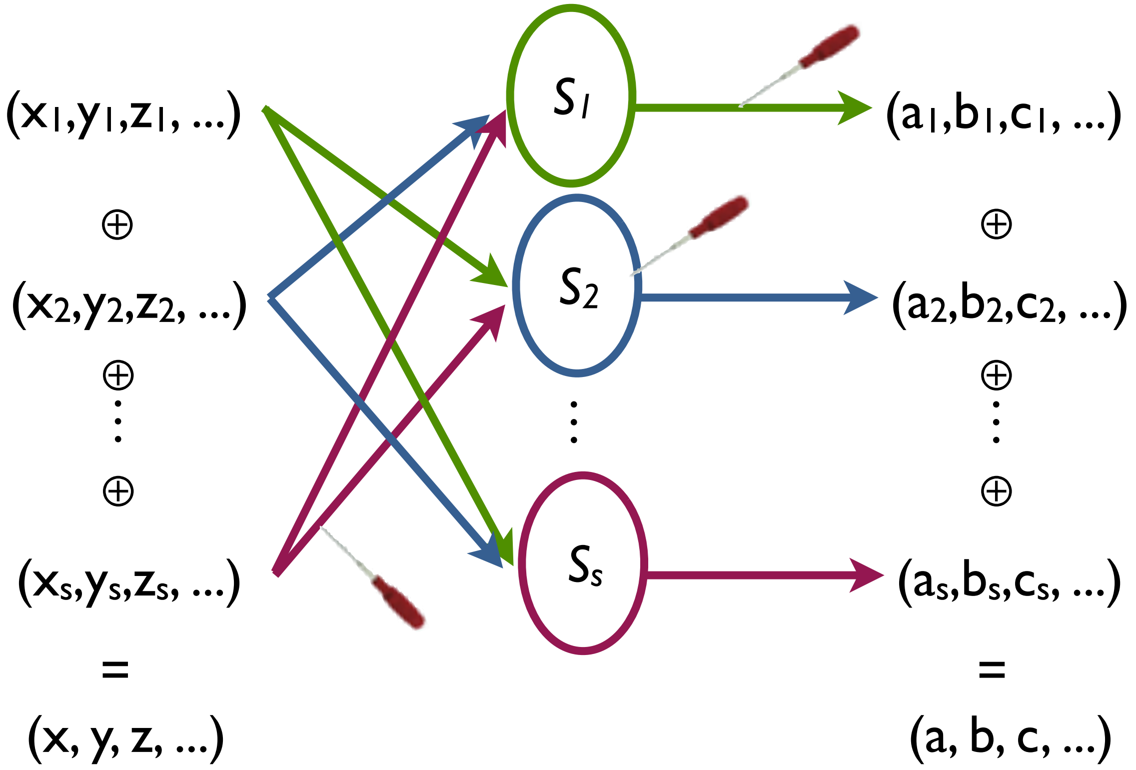
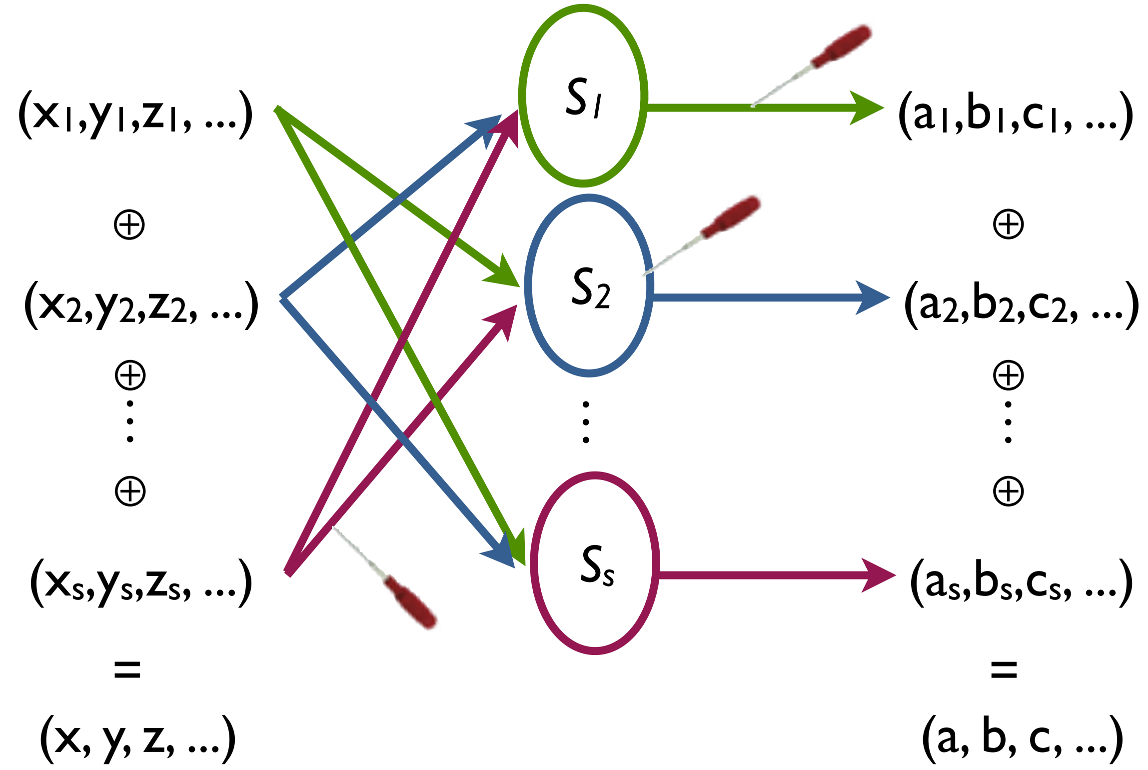


Uniform input masking

Correctness

Non-completeness: Every function is independent of at least one input share.

Higher-Order Threshold Implementations



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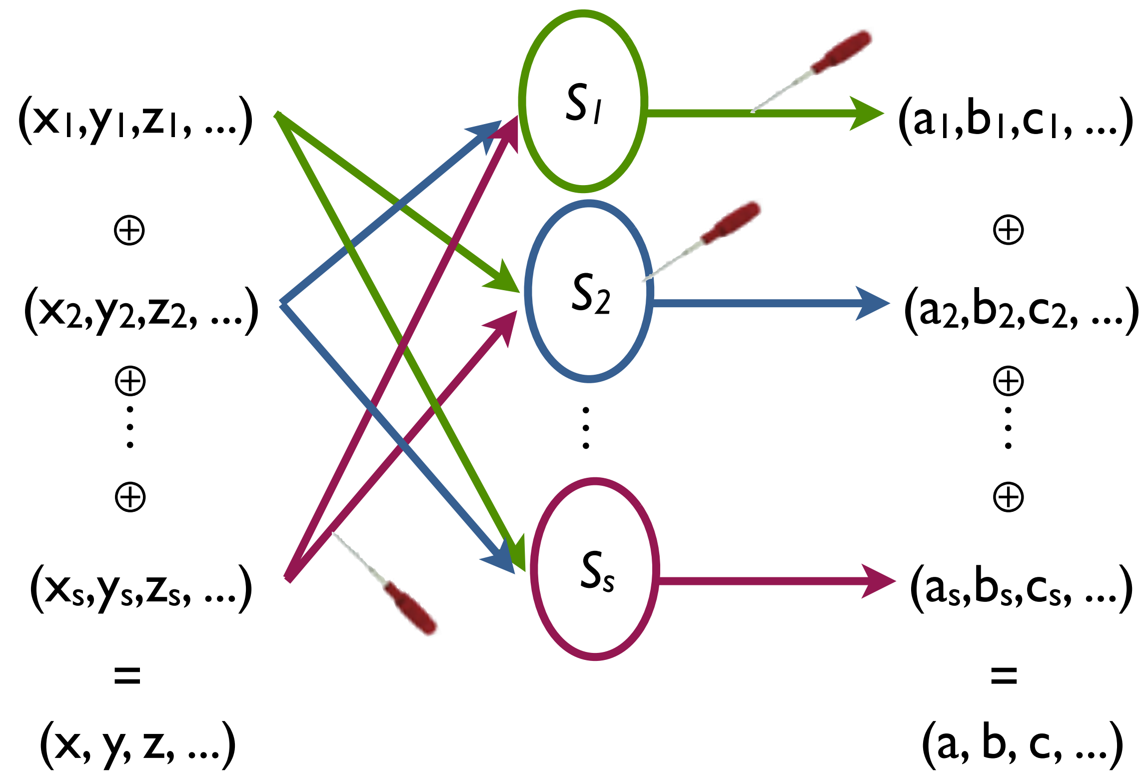

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How many shares are necessary?

Higher-Order Threshold Implementations

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Linear functions

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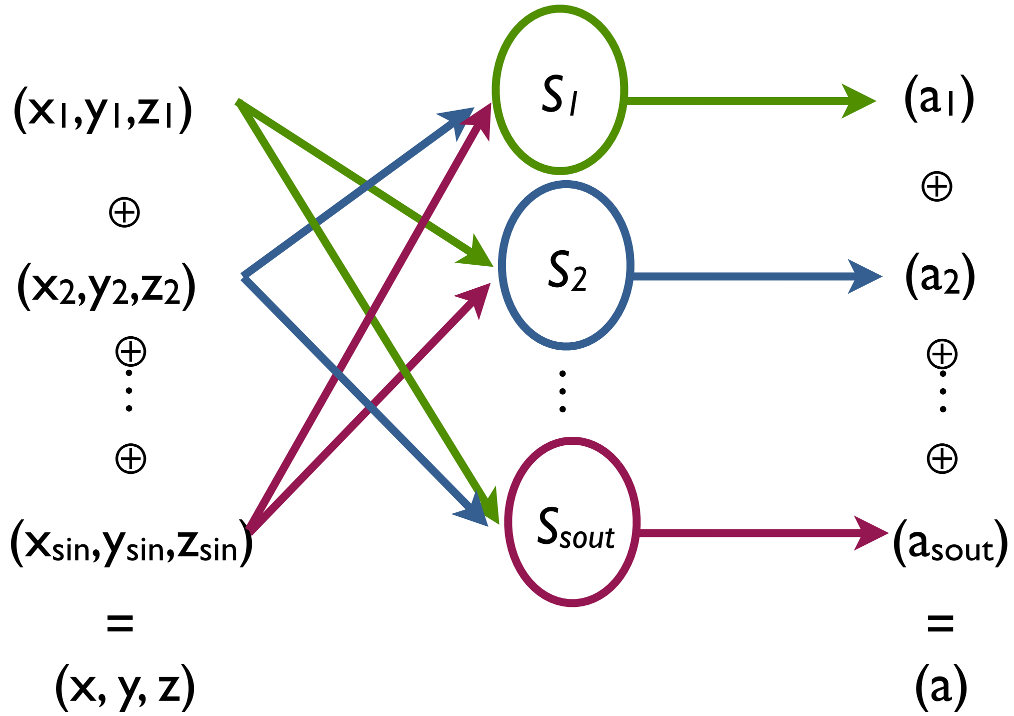
Higher-Order Threshold Implementations



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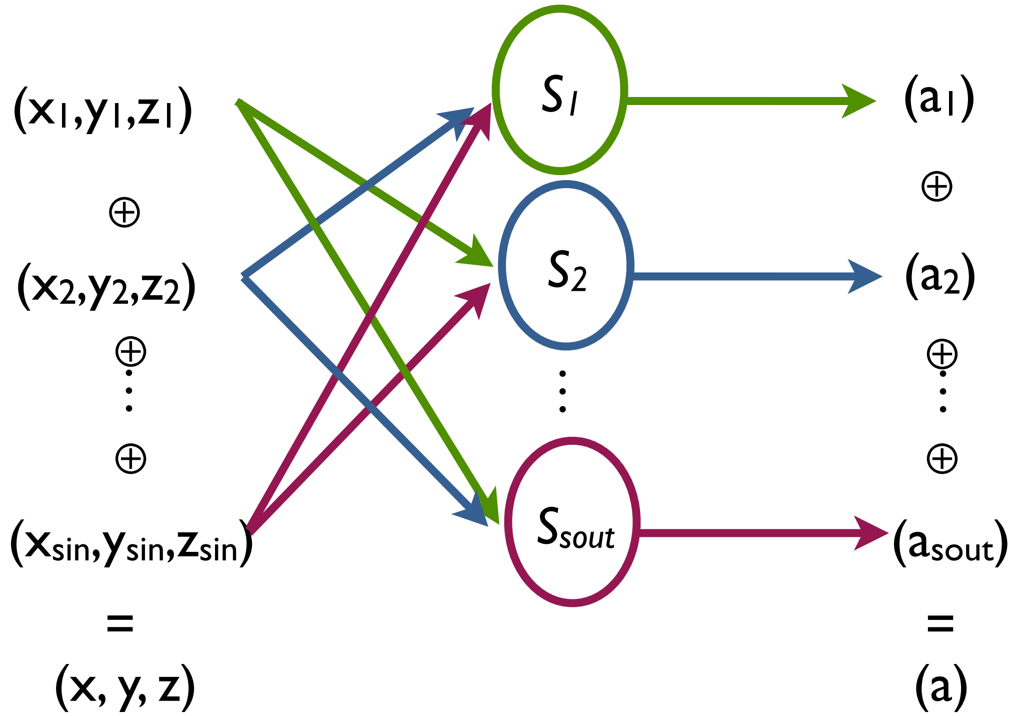
Nonlinear functions (a = $S(x,y,z) = xy+z$)

- More challenging

Higher-Order Threshold Implementations



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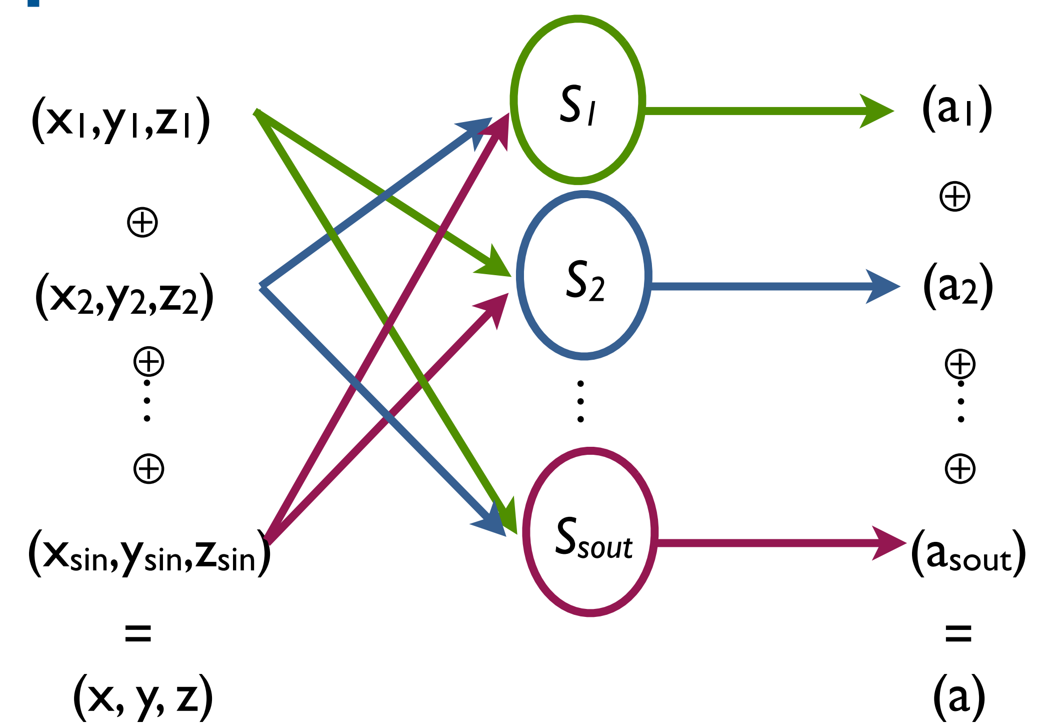

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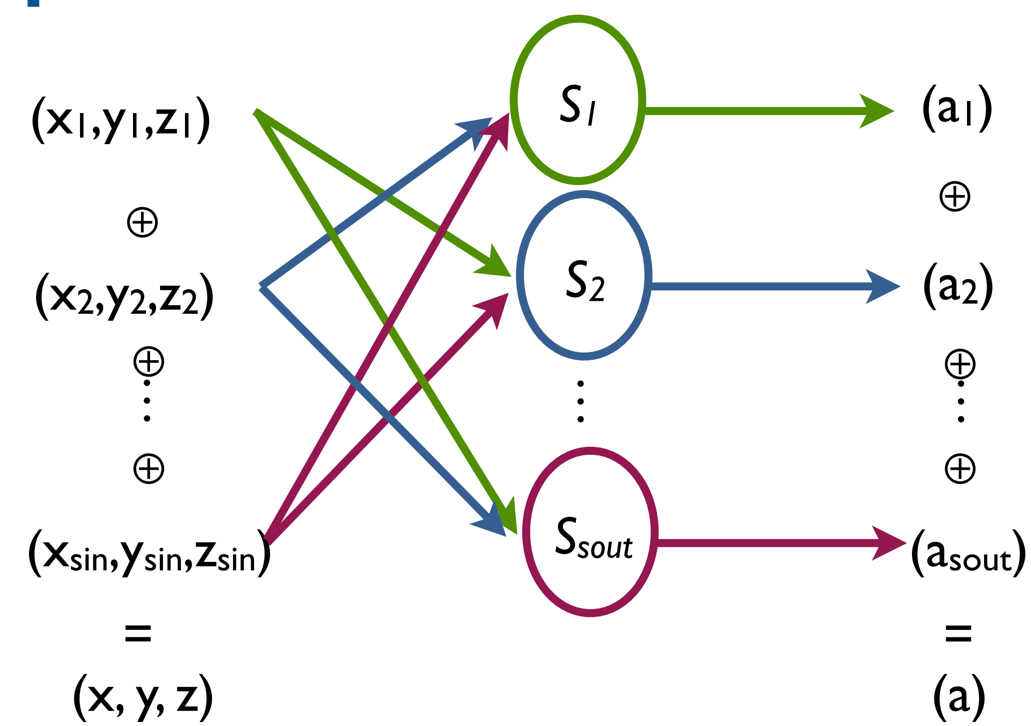

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- First-order $s_{in} \geq 3$ input $s_{out} \geq 3$ output shares
- Second-order $s_{in} \geq 5$ input and $s_{out} \geq 10$ output shares

Higher-Order Threshold Implementations

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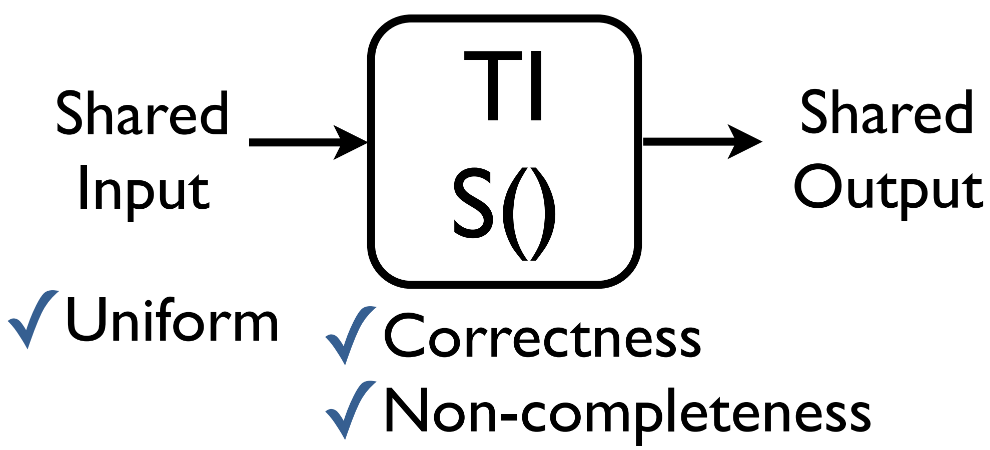

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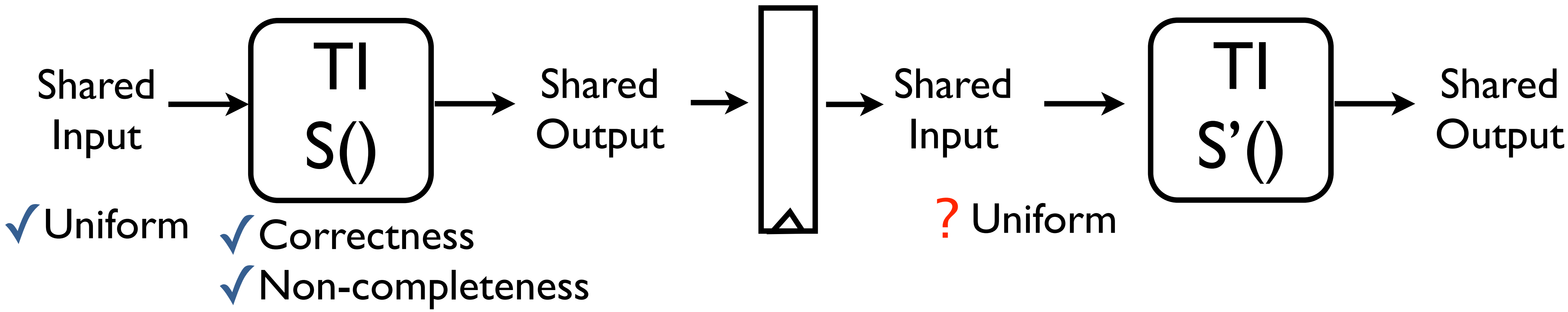

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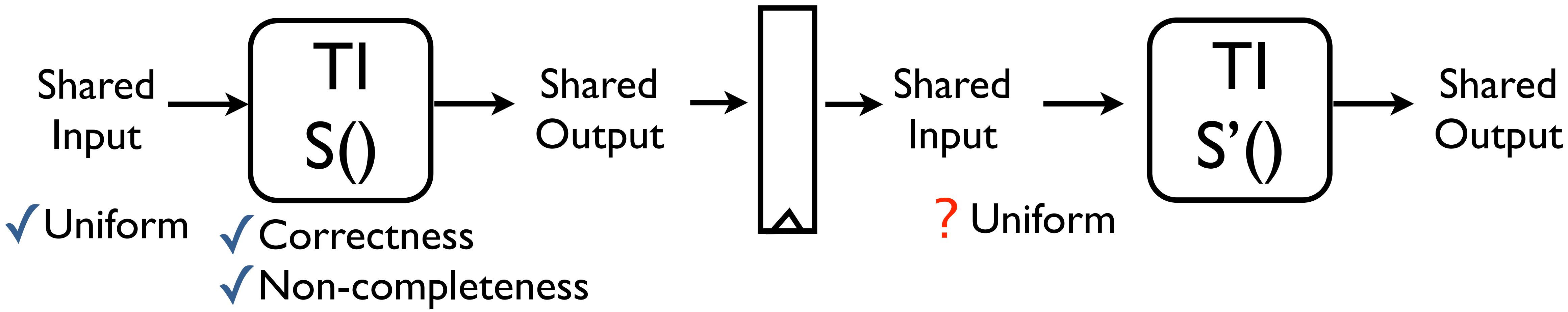

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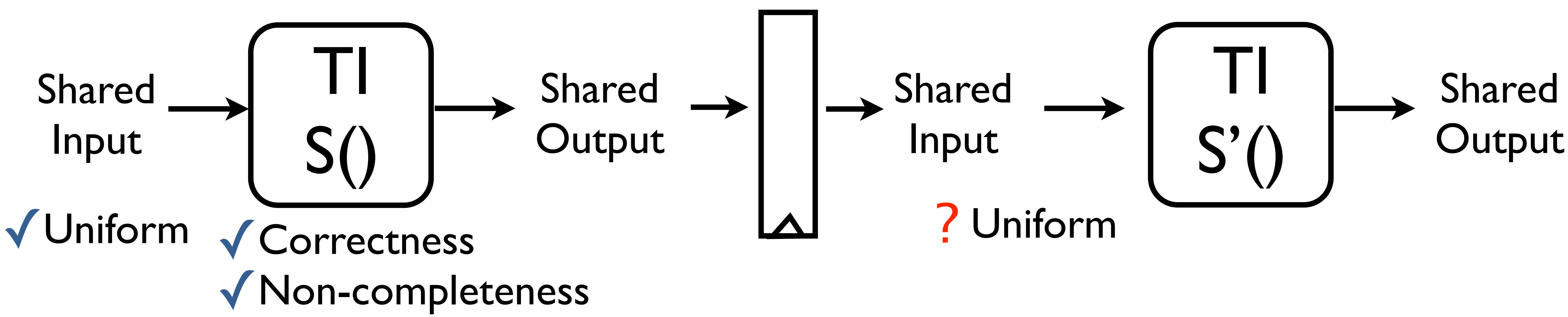


One solution: XOR some of the output shares

In our paper:

- Uniform HO-TI of an AND/XOR gate
- Uniform second-order TI of quadratic 4-bit permutations

Higher-Order Threshold Implementations

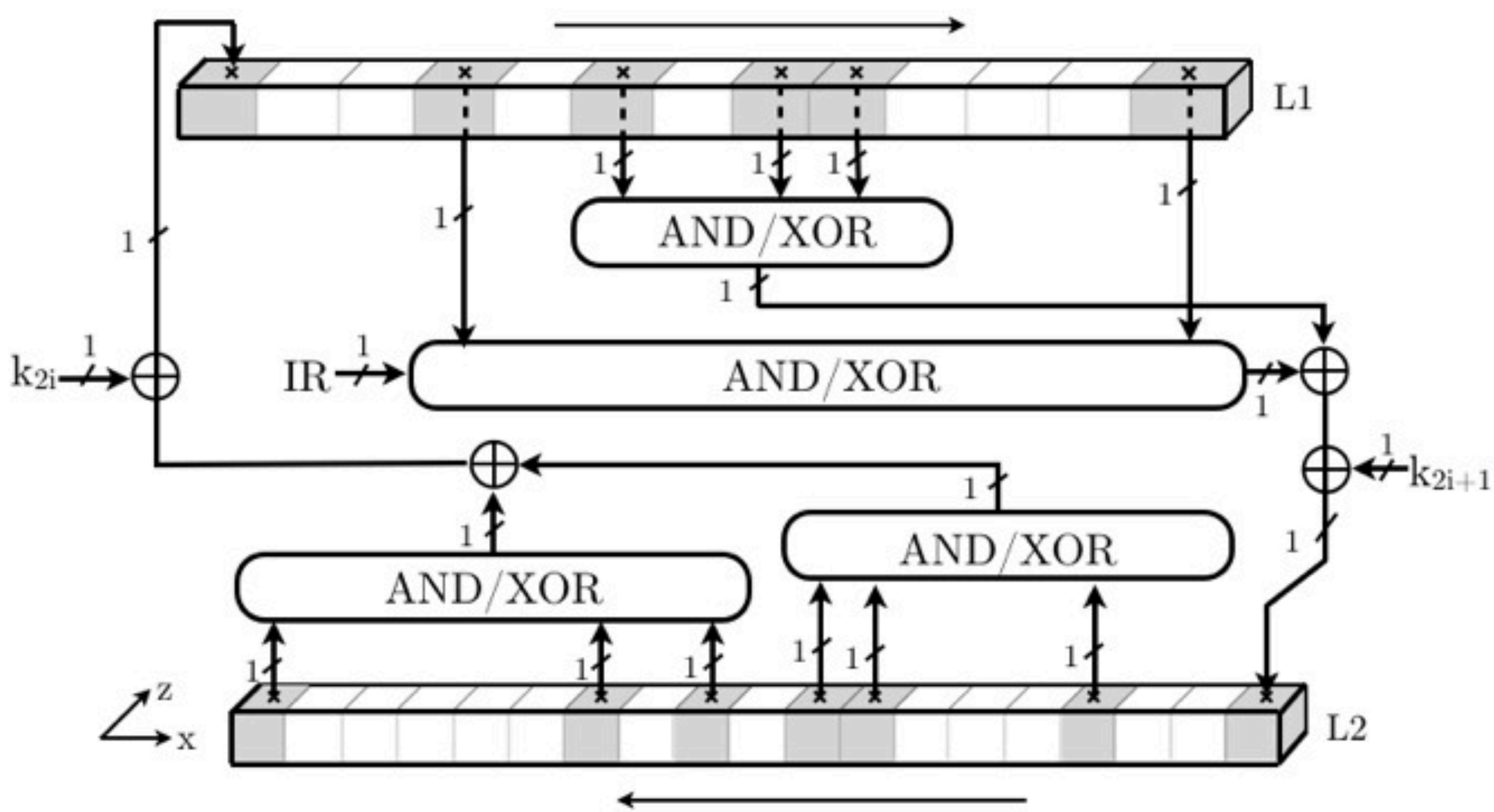
Application to a cryptographic algorithm
&
Testing

Higher-Order Threshold Implementations

Second-order TI of KATAN-32
&
Leakage Detection Tests on SASEBO-G

KATAN-32

- 254-round block cipher
- 32-bit plain/cipher-text and 80-bit key
- Round keys are generated by an LFSR



Linear: $s \geq d+1$
Nonlinear: $s_{in} \geq td+1$ and $s_{out} \geq \binom{s_{in}}{t}$

HO-TI of KATAN-32

	# of shares		
	Linear	Nonlinear	
		s_{in}	s_{out}
Unprotected	1	1	1
First-Order TI	3	3	3
Second-Order TI	5	5	10
Third-Order TI	7	7	21

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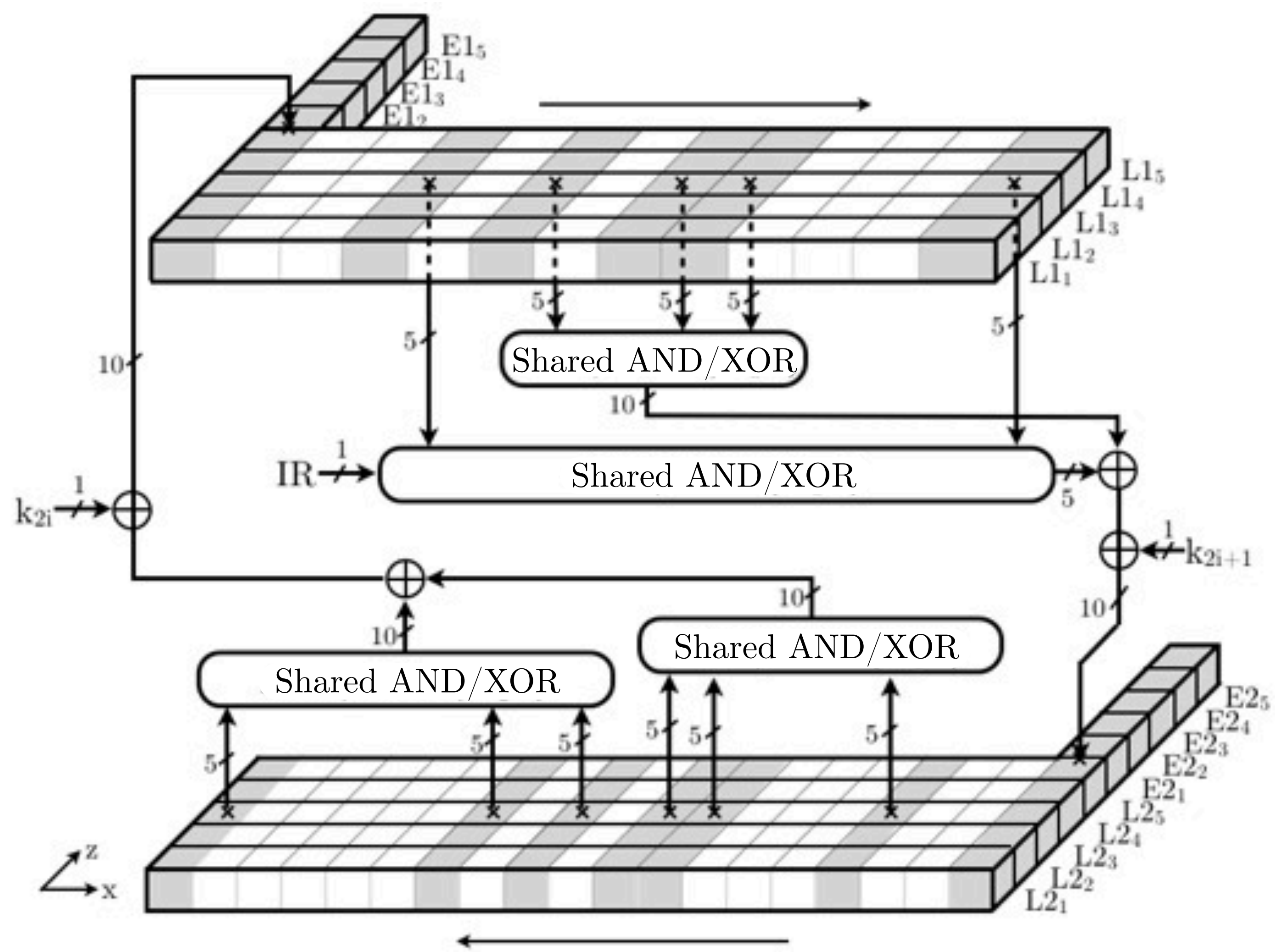

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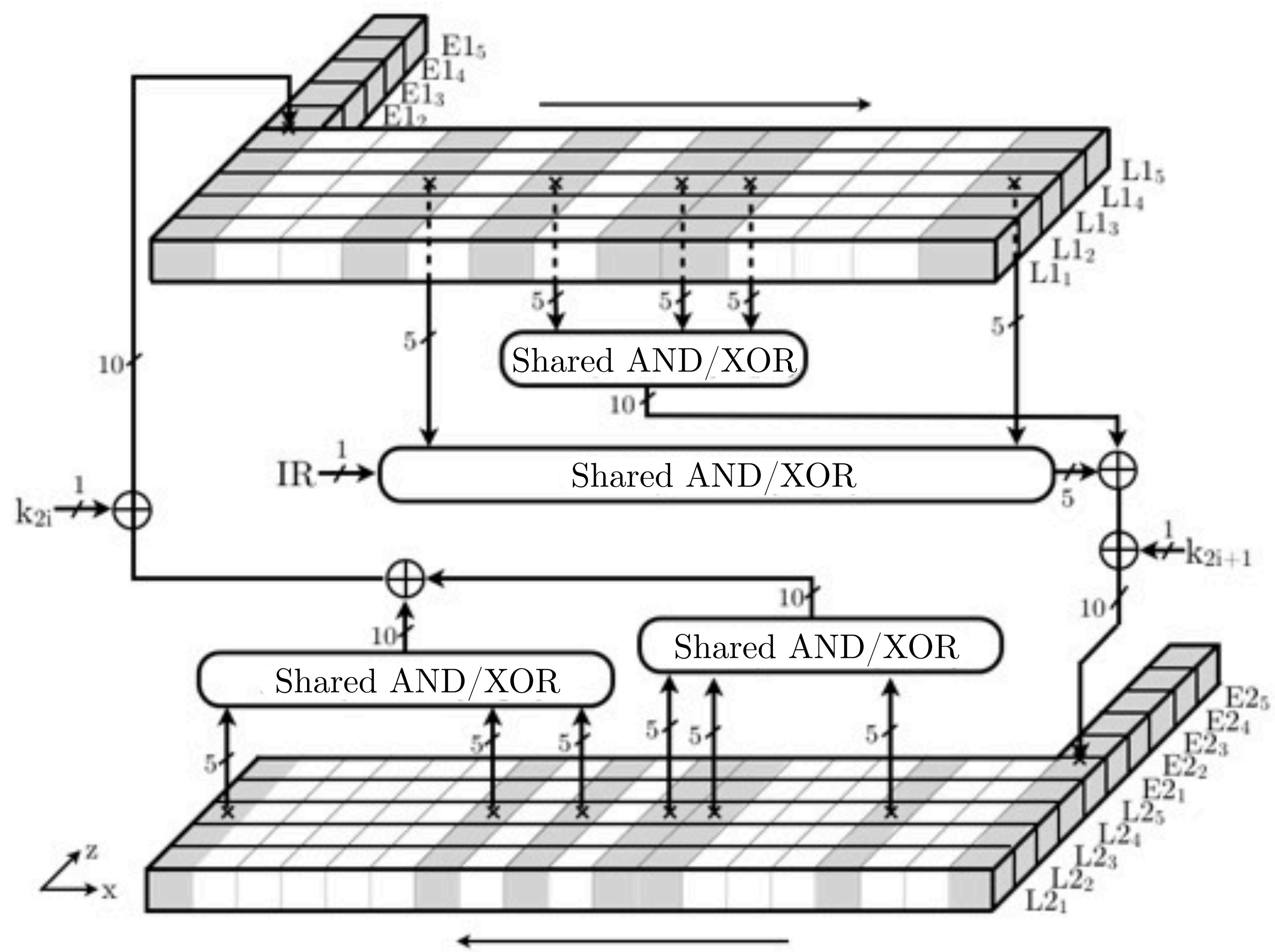
HO-TI of KATAN-32

	# of shares			Area (GE) Faraday Standard Cell Library FSA0A C Generic Core
	Linear	Nonlinear		
		s_{in}	s_{out}	
Unprotected	1	1	1	1002
First-Order TI	3	3	3	1720
Second-Order TI	5	5	10	2556
Third-Order TI	7	7	21	3539



HO-TI of KATAN-32 - Analysis

Fix vs. random leakage detection test
RNG is OFF to test the setup

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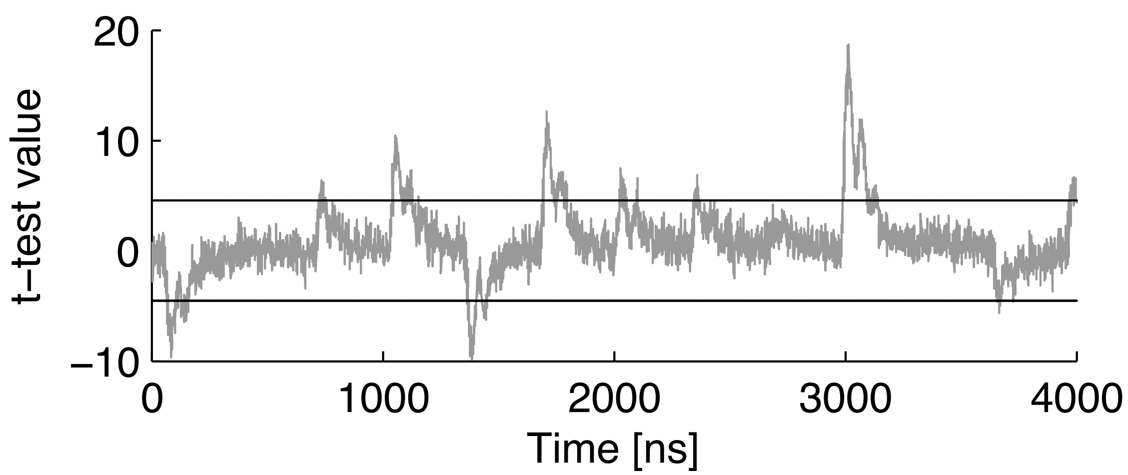
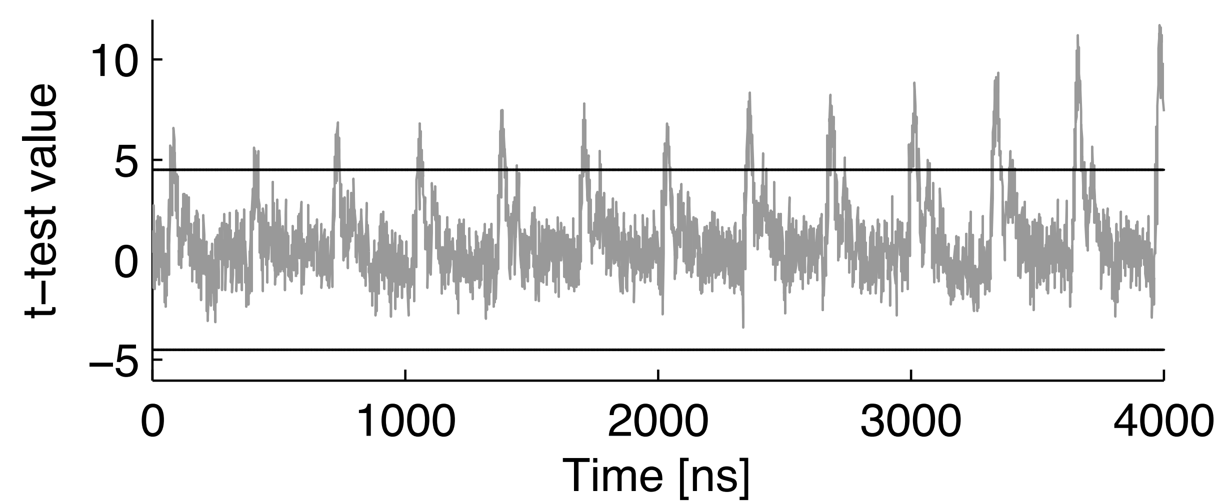
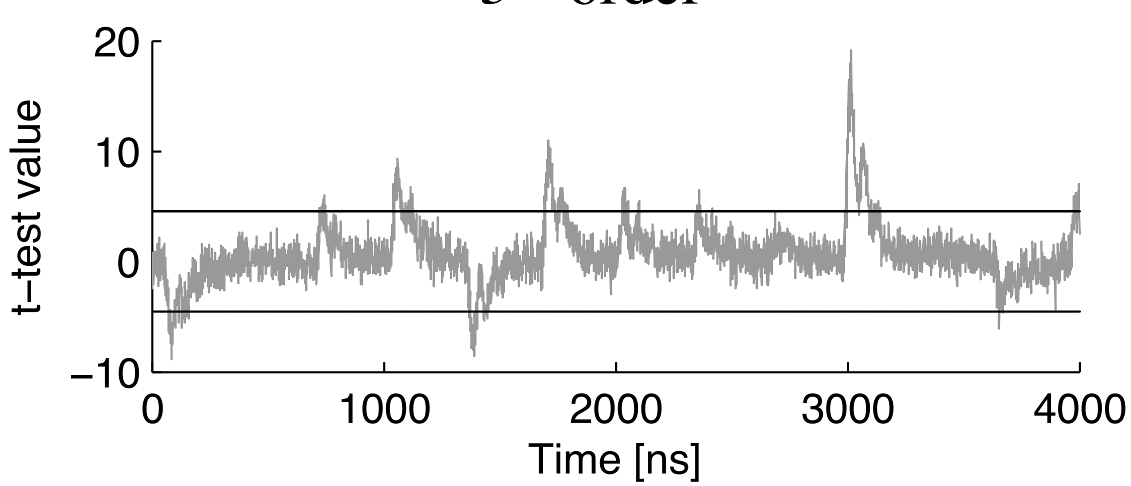
1st-order



2nd-order



3rd-order



1000 traces

HO-TI of KATAN-32 - Analysis

Fix vs. random leakage detection test
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HO-TI of KATAN-32 - Analysis

Fix vs. random leakage detection test

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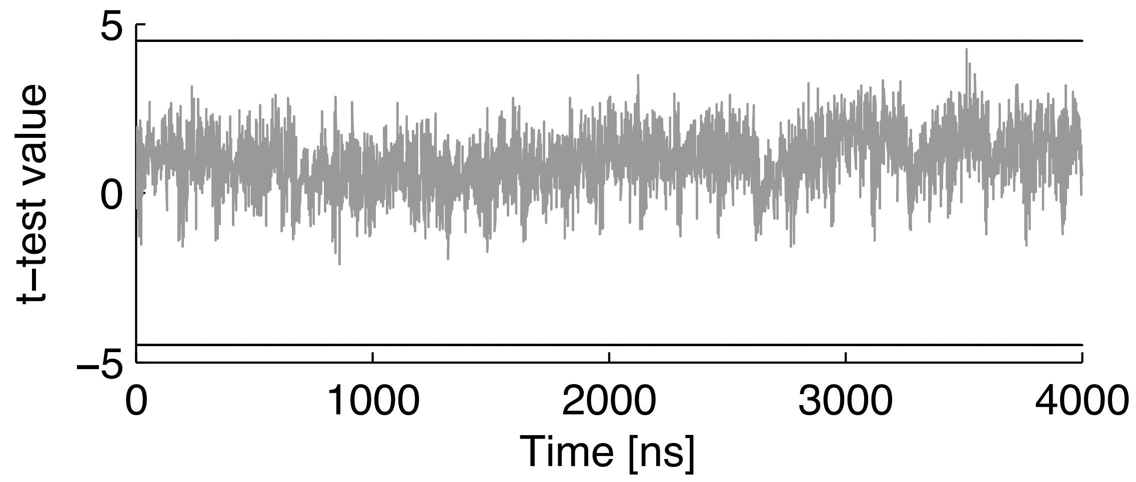
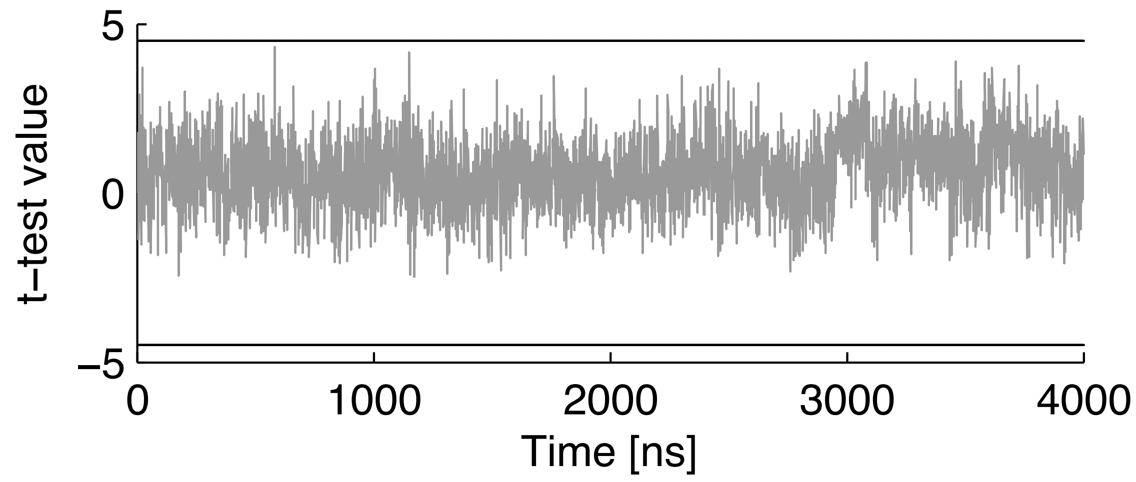


2nd-order

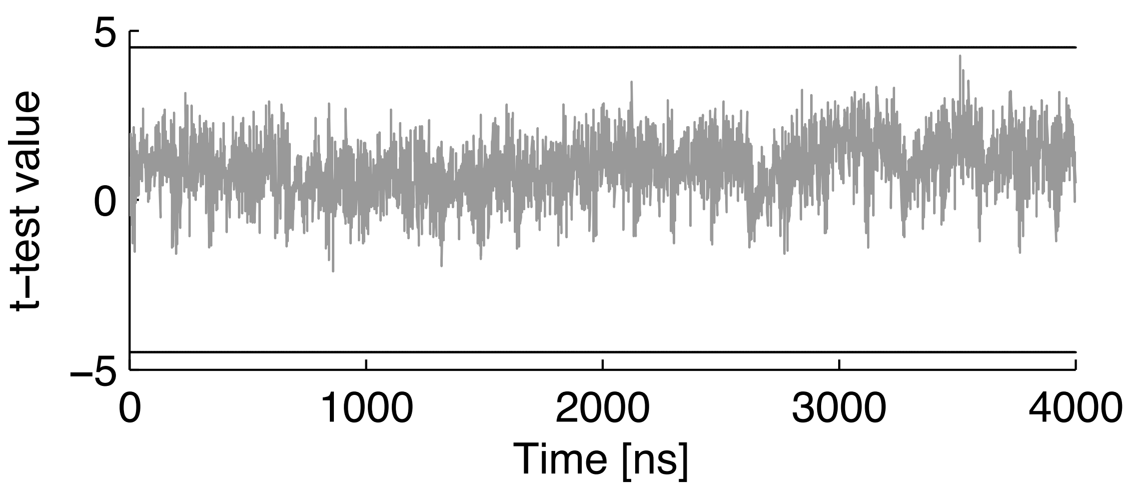


HO-TI of KATAN-32 - Analysis

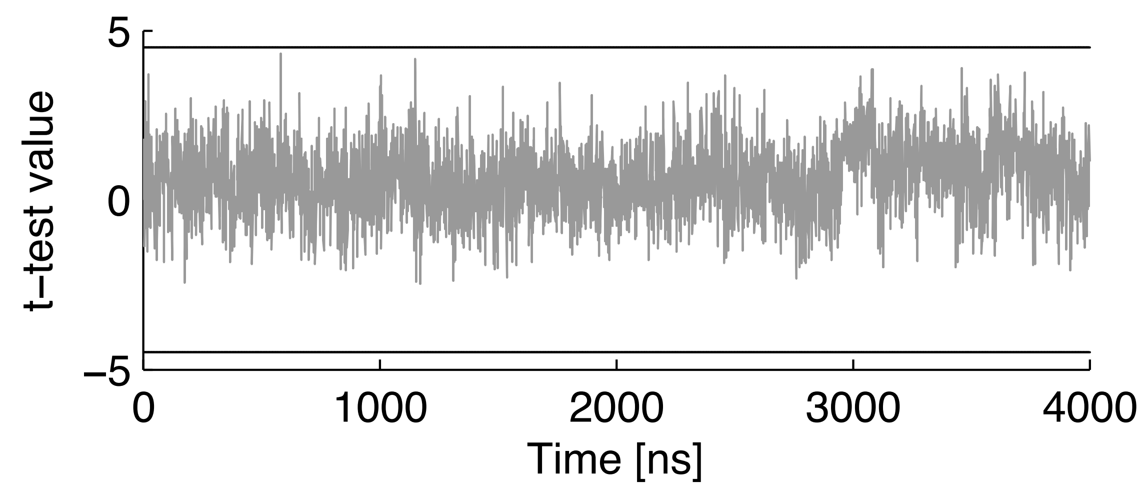
Fix vs. random leakage detection test

RNG is ON

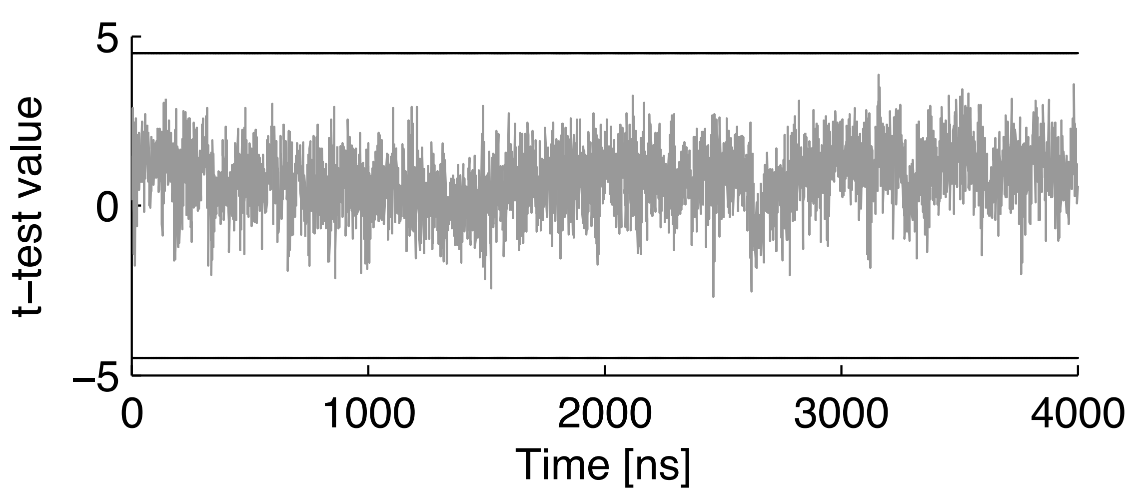
1st-order



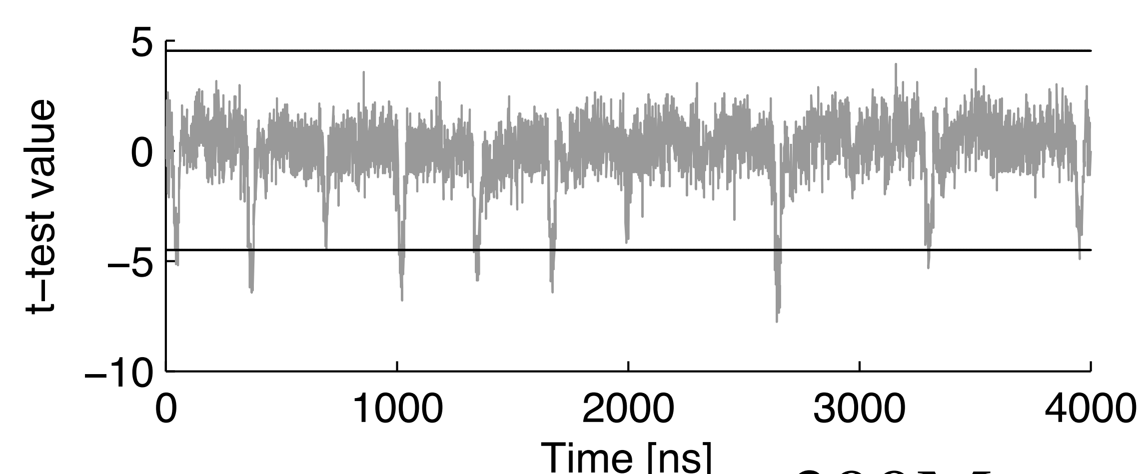
2nd-order



3rd-order



5th-order



300M traces

Conclusion

- Countermeasure against HO-DPA
- Efficient TIs of KATAN-32
- Confirmed the claimed security using leakage detection tests
- Methods for second-order TI of quadratic 4-bit permutations

Thank You!